ASPECTS OF PSYCHOPEDAGOGY OF "AXIOMATIC METHOD" AND OF "CONSTRUCTION" IN MATH THINKING

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Abstract: To increase the tasks in instructive educational of mathematical thinking, but also to avoid any confusion, we have, on the one hand, to distinguish between "axiom" and "axiomatic", on the other hand, to make a explanatory distinction between the concepts "construction", "contructivism" and" building ".

The verbal meaning of the word axiom is a self-evident proposition that no longer requires demonstration, but another system can be a simple theorem. Our approach takes into account the absolute sense of the concept of "axiom", but also at relatively specific current logic (axiom in S-S is a axiomatic system), is "the axiom reductibility" made by B. Russell, but also "axiom syllogism" ("what they say about all or denies, denies or says about each one"). Clear awareness of these issues increases the sense of more efficient learning of mathematics.

On the other hand, clarification on the meanings mentioned above the concept of "construction", promotes education type mathematically by assimilating the psychological meaning of "standard" by which directs the "construction" as an activity during which the focus is on structural elements to be the result of creative activity.

One who noticed this is Z. P. Dienes and bring back whatever it possible criticism. In other news, constructivism, viewed from the perspective of psychology of learning, is a variant of intuitionism logical-mathematical logician Romanian where Gh. Enescu is said that the fundamental notion of "algorithm".

From the psychological point of view it is not necessary activity to occur simply, but it is imperative that it takes place efficiently, is to provide the possibility of expressing creativity, any specific mode of learning, especially mathematics. Progress can not be achieved only through a

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creative learning, Constructivist specific algorithms suitable. Therefore, Z. P. Dienes noted that constructive side of mathematical thinking (November 1 and the creative add) occurs only when you need to develop a structure, knowing its nature and properties. Thus, we can avoid handling, ignorance, failure, failure and limiting creative thinking. In this way the essence of mathematical object becomes a potential building feasible, and the truth, as said, Gh. Enescu, is identified with "verified as true."

*Keywords:*axiom, axiomatic construction, constructivism, construction, creative thinking.

1.Pedagogical aspects of axiomatic method

To avoid confusion about the *axiom* rice and *axiomatic* offer a sneak peek into the field. The verbal meaning of the word *axiom* is a self-evident proposition that no longer requires demonstration, but that is not necessarily obvious and that the other system can be simple theorem. Traditionally there is sometimes accepted axioms in *an absolute sense* (as evident indemonstrable sentence, while in terms of actual logic axioms have only *relative sense* ("axiom in S"- S is a axiomatic system).

The literature speaks of *reductibility axiom* formulated by B. Russell, according to which any sentence can unpredicative predicative formula one equivalent; *axiom syllogism,* as the fundamental law of syllogism expressed briefly in Latin by *the dictum of omni et nullo* (say about everyone and about none). In other words, "what is said all is said and about each one, what about all deny and deny each appear about you"[1].

Axiomatic method is a systematization of sentences a range of information and involves the following principles:

a) postulates a finite number of terms (terms) appointed *its first (raw terms)* and other rules of definition of terms, *terms* called *derivatives (derived notions)*;

b) postulates a finite number of *axioms* and propositions called raw inference rules of other sentences called *time.*As a result give an *axiomatic system.* If applied to *objectsformal,* then we get a *formal axiomatic system*[2].

The most common seem to be *axiomatic propositions* specific Hilbert system-Ackerman, system logic functions truth *Peano axioms,* fundamental propositions formulated by

[1]Enescu, Gh., *Op. cit.* , p. 25.
[2]Ibid, p. 26.

Italian mathematician of the same name for the natural numbers and arithmetic *axioms* of Zermelo and Fraenkel formulated *crowd.*

In short, Peano's axioms are:

1) 1 is the natural number;

2) the successor of any natural number is also a natural number;

3) no two numerte same natural successor;

4) 1 is not the successor of any natural number (in other cases it took 0);

5) if one (or 0) is a property and if the fact that *I* have this property that his successor has this property, then any natural number has property.

Proposition (5) is given by Gh. Enescu as the substantive *principle of mathematical induction*[3].

Peano uses only three raw arithmetic concepts: *natural number, zero (or one)* and *successor.*At first sight You might say that these concepts are defined *implicitly* by axioms, but the possibility of replacing *one* with *zero* shows that things are not so. As shown by Th. Skolem system extends them by extending the notion of *ownership* and is *monomorphic* only if the terms of the *crowd* and *function propositionalprinciples* are taken independently of any *generation.* To avoid this generalization Cantor's paradox we must put in place the axiom (5) a lot of axioms from which each relate to a *property determined.* If we introduce a countable infinite number of axioms, the system is *multiforme* (necategoric) fully[4].

The literature has shown that axiomatic method specifies *responsibilities* mathematical theorem proving. Thus, students in mathematics, as distinguished RI Wilder says before a demonstration that could make if they knew what it may entail.

Axiomatic method essentially facilitates the demonstration because the statement "will demonstrate a Σ - statement "(ie a statement axiomatic system belonging Σ) expressly stated that the assumptions which may be raised in the demonstration are formulated in the axioms of Σ; therefore axiomatic method establishes responsibilities and powers. The student thus having a specified direction in addressing the demonstration, how would specify RJ Wilder, therefore

[3]Ibid, p. 28.
[4]Skolem, Th., *Selected Work in Logic* , edited by JE Fenstad, Universitetvorlaget Oslo, Bergen, Tromso, 1970.

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axiomatic method involves *pedagogical relevance*, hence the ways a case for extending it into education[5].

Besides the positive aspects, the literature reveals the limitations and disadvantages of the axiomatic method. Critics reveal its *dependence on logic "logic* vulnerability is transmitted and the axiomatic method". He has been accused of *high character* poi *formal* axiomatic method, which can be applied, ignoring any interpretation; by D. Hilbert, consistency of axiomatic system is controlled by invoking a pattern: non-Euclidean geometries are consistent if Euclidean geometry is consistent.

In this regard, A. Heyting[6]state that the axiomatic method in mathematics background is unfit to autonomously, always asking her results interpretation Extra- axiomatic, except the view expressed by D. Hilbert's program. But it not only he hoped a reconstruction axiomatic basis of mathematics; it is Axiomatization of some branches of mathematics. There are, however, says Marin Ţurlea, and mathematicians who refuse this ideal, stating that mathematical thinking has to do with axiomatization and formalization which instead to return substance mathematics, a sacrifice[7].

In turn, ELJ Brouwer criticized axiomatic foundations, challenging systems logical - linguistic any role in the foundation of mathematics[8]. In *terms ofpsychopedagogical* argued that formalizing fundamentals is a waste of time and that this method is inadequate learning insufficiently mature minds, as claimed and ZP Dienes on abstraction.

System consistency of logic, not that there is a system thematic me, ELJ says Brouwer. Axiomaticiens, he says, they have demonstrated this nor that a finite number must always exist. He noted, "the assertion certainly is not true if the circumstances do not imply that the system should be constructive. For example, according to Hilbert, properties that Cantor has formulated crowd well - ordered all the numbers in the second class numbers are consistent, but the crowd there is in mathematics"[9].

Regarding the so-called *"axiom of infinity" (there are infinite sets),* Marin Turlea appreciate that it "transcends the everyday practice or scientific experience and therefore the

[5]Wilder, RI, *Introduction to the Foundations of Mathematics* , New York, 1956, p. 43.
[6]Heyting, A., *Les fondements des mathématiques. Intuitionism. Thiéorie the demonstration,* Gauthier - Villars, Paris, 1955, p. 38.
[7]Ţurlea, M., *Philosophy and foundations of mathematics* , Bucharest, Academy Press, 1982, p. 15
[8]Brouwer, ELJ, *"On the Foundations of Mathematics"* in *Philosophy and Foundations of Mathematics* , the Collected Works, vol. 1, North - Holland, Amsterdam, 1975.
[9]Ibid ,, p. 8.

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mathematical"[10].The problem here may thus be formulated: *that mathematical objects are permissible.* The research reveals two philosophical attitudes: *Platonism,* which states that any mathematical objects exist as objects, and *constructivism* which in mathematical terms, proposes slaughtering an important part of mathematics[11].

K. Gödel developed a *realisticfilosofie* mathematics. In the philosophy of mathematics, *realism* is the idea that the laws of mathematics are literal descriptions of the objections of some sort. Realism gödelian is "a particular form" in senul the *crowds* and *the views* are not localized in space and time, but are beyond our control, but" aprehendate and described by us, opposing *nominalism* (= no abstract entities, non - space) and also *conceptualism* considers that there is also abstract entities, but through the work of our mental math to sell the subject of the eternal objects non - spatial, but real[12].

Directly or indirectly, realistic philosophy of mathematics has been criticized for several reasons. If we consider a work of Stephen F. Barker, expose briefly following objections to realistic philosophy of mathematics:[13]

-First, it was objected that it is too *metaphysical* realist philosophy *as* a field of entities postulated non - empirical, abstract, independent and accessible only existing human reason; experience does not provide, therefore, no evidence here[14];

-The second complaint relates to the disastrous consequences of *the theorem of Gödel:* we cannot assume a field goal real, independent of our minds, as the subject of mathematics if we realize that no formal theory complex and consistent - a situation in which it is set theory and number theory - cannot be built.

In this regard, Marin Turlea reveals that *Gödel's Mathematical Logic* in *Russell*[15] shows that the theorem does not question the *ontology of mathematical entities,* as was thought, it only indicates a limitation in the *"power of expression"* formal systems that can *capture* objects symbolic system like that of the natural numbers; "Is not realistic that the philosopher will

[10]Ţurlea, M., *op. cit.,* p. 21.
[11] Ibid.
[12]Gödel, K., *"Russell's Mathematical Logic"* in Benacerraf, P., *Philosophy of Mathematics* , Prentice Hall, 1964.
[13]Barker, SF, *"Realism as Philosophy of Mathematics"* in *Foundations of Mathematics Papers Symposiuim commemorating the Sixtieth Birthday of K. Gödel* , Springer - Verlag, Berlin, Heidelberg, New York, 1969.
[14] These objections are noted Marin Turlea in *philosophy and foundations of mathematic,* Bucharest Academy Press, 1982, p. 22-24.
[15]Gödel, K., *"Russell's Mathematical Logic"* in Benacerraf, P., *Philosophy of Mathematics* , Prentice Hall, 1964.

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demonstrate his belief in the existence of numbers, classes (sets), morality is just that you can not make a list of axioms that give a complete description of the universe of mathematical objects[16].

-The third objection against realism belongs critique P. Benacerraf did logicism, indicating the outside of ways to define the crowds particular numbers, which means that the numbers are not sets, their characteristic appearance representing it in essentially, *recursive progression;* formulating their essential properties means the return to stating an *"abstract structure",* which attacks the consideration of numbers as objects.

Critics under the influence of P. Benacerraf, realist will have to admit that the numbers do *mathematical objects* constitution for his *fundamental* and philosophical joins the idea that set theory is *"a fundamental part of mathematics",* the crowds are real objects in the sense as defined above; In this case, adept realistic philosophy of mathematics will operate a *reduction* of number theory to set theory. P. Benacerraf likes to emphasize that the most important number system is *"structure"* than "subject".

-SF Barker stated last objection refers to a situation of set theory, namely *to what extent it is legitimate to consider crowds as objects;* firstly there is no single theory of sets; then, we cannot determine which of the alternative forms of the theory of sets is inconsistent and that is true; Finally, it would make sense to assume that an area void of objects - crowds - is the specific *topic* of mathematics because we cannot give a meaning.

Regarding these issues, Marin Turlea opinion that, currently, active mathematicians ignore the philosophical aspects of mathematics, preferring "to deduce theorems from axioms imposed by the authorities." However, philosophical activity can stimulate scientific research program and proposing optimal models, conceptual classification issues often preparing solutions. Discussions on matters ontological, logical, psychological, pedagogical and methodological philosophy of science today are treated as internal affairs of specialists in the field.

Subject genuine belonging to *fundamentals of mathematics* is the *nature* and *significanceofmathematics.* These topics have generated controversy explained by the fact that the overall development of Boole, Cantor, Dedekind, Schröder, Frege, Weierstrass, Peano,

[16]Ţurlea, M., *op. cit.,* p. 2. 3.

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Zermelo were accompanied early twentieth century fundaţioniste programs that have assumed "rival philosophical and competitive"

- *Logicism,* developed by B. Russell, who proposed the unification of mathematics by reducing its shares in the most basic and general parts, *logic* and concept of *the crowd;*

-*Formalism* (David Hilbert's program), which Absolutized study symbolic structures of mathematical language, trying to explain their attitude in terms of synthetic rules whose justification was to be guaranteed by demonstrations accordingly;

- *Intuitionism,* merely to advanced Kronecker and H. Poincaré, but also some ideas of Kant; stated radical renunciation of the use of language and logic classical and demanding a severe limitation of the types of allowable mathematical construction[17].

2."Construction" in math and its educational aspects

In our opinion, to increase the chances educational aspects of mathematical thinking, we must make a distinction Explanatory between *"building", "constructivism"* and *"building".* In the strict sense, *"building"* lies in the composition of things to achieve other structure after further specified certain requirements, which may be more or less mandatory or specific.

In the words of the F. Bartlett, they are *"standard"* which is oriented to construction[18], activities in which you focus on *the kind of things* that to be the end result of this activity. It is not about real things and therefore, for the activity to be successful, "must all classes used in the construction to be well established", as expressed ZP Dienes.

He stated that any given copy of the class for which the defining attributes are relevant or significant - that any element of the universe assertion - will be known as belonging to that class or not. From the psychological point of view it is not necessary that the activity takes place, but it is probably necessary for it to occur effectively[19].

Constructivism is a variant of intuitionism logical - mathematical developed by IU School AA Makarov, and in some cases the term is taken as synonymous with "intuitionism logical - mathematical". After his explanation Gh. Enescu, constructivism is fundamental to the notion of

[17]Ţurlea, M., *op. cit.*, p. 25.
[18]Bartlett, F., *Thinking* , London, Allen and Unwin, 1974.
[19]Dienes, ZP, *op. cit.* , p. 62.

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algorithm; the essence of mathematical object construction is potentially achievable; the truth is identified with *"verified as true"*[20].

According to ZP Dienes, constructive side of mathematical thinking occurs when we have to develop a structure, knowing, somehow, type. This kind of thinking is practiced by artists: he does not know exactly what picture to paint but, in the words of F. Bartlett, has in mind a precise idea of the type of picture you want to paint it[21].

He proved as reliable judgment fails when trying to live up to his self proposed, though perhaps I will find it difficult reasoning behind the judgment. ZP Dienes argues that the origin of mathematical constructions must be sought in *mathematical game manipulative;* "What we call a building is largely arbitrary, since virtually any building can be developed in another continuation or can be separated from another, and it would be hard to say where one ends and where it begins building another. Abstraction has a constructive character par excellence "[22].

If we start from certain elements and then build a class (in the mathematical sense) become more or less aware of that class structure and class element properties. Develop mathematical entity, ZP Dienes stresses that show some given property has a constructive character. He cites the *theory of functions of a real variable:* if we take an infinite sequence of continuous functions, *limit function,* if any, may or may not continue. If it's not continuous, we can call *discontinued function* of class 1, class functions continue being zero. Such a function would be, for example, limit the function of sequence $X^{n,}$ taken between 0 and 1 inclusive; thus n ∞ limit is the function that takes zero between x = 0 and x = 1 but is equal to 1 for x = 1. The question arises whether there are discontinuous functions which do not constitute the boundaries of continuous functions; and if there are, we wonder whether all functions constitute limits of Class 2. One wonders if there is no function of class 3.

Examining the analytical aspect of the idea of function in general, says ZP Dienes, I can answer that would be impossible for such functions do not exist. "This would be an analytical solution, not constructive. "A" constructive solution should pursue *"actual construction* of a function to be limit functions of class 2, without itself grade 2 or lower. A function of the class 2

[20]En escu, Gh., *Op. cit.* , p. 53.
[21]Bartlett, F., *op. cit.* , p. 18.
[22]Dienes, ZP, *op. cit.*, p. 97.

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is admitted, for example, a value of 1 for all values of irrational and 0 for all reasonable values "[23].

This can be checked practically verification would coincide with the analytical aspect of the construction which must be held before the discovery that the function is class 2. Another example is the mathematical *construction* Peano curve, which is a continuous function Why go through all the points of a square. *the requirement of continuity* and that *of going* through all the points of a square are met by building such a function.

Another example more understandable gives us O. Becker on the way the ordinal numbers can be generated string trans - finished with two generating principles in the minds of G. Cantor. The two principles are:

1. Each number, a date format and add a unit, the first ordinal number one.

2. Each sequence of numbers predefined sequence in which no other element is assigned a special number created, which is designed to limit numbers belonging succession, defined, therefore, that the greater number immediately following.

G. Cantor reached by analyzing the *"first epsilon number"* and introduce a new symbol **ε** which says it is a *"critical mass."* I n an attempt to note trans ordinal numbers - finished by signs symbolic always meet such numbers critical, which forces us to change the method of notation, ie the introduction of new symbols.

But G. Cantor goes further and applying the second principle of inheritance of all trans ordinals - finished the second class (the ordinals and countable infinite powers that cannot be clearly defined by any single constructive process)[24], get to "create" the first ordinal trans - finished uncountable ordinal number Ω or **ω** starts with the third class of ordinals. Thus he manages to obtain class ordinal rank of increasingly higher.

In the classical theory of sets were used *"special"* introduced with the two generating principles: it is E. Zermelo, D. Hilbert and Gödel K.[25]. The example it uses O. Becker is a method of construction that comes from GH Hardy, in a simplified form, the Hausdorff F.[26]. In this example embodiment mathematicians saw a "constructive" a well ordered and uncountable

[23] Ibid.
[24] Becker, O., *op. cit.*, p. 159.
[25] Details in O. Becker, *op. cit.* p. 159.
[26] Hardy, GH, *Quarterly Journal of Mathematics* , 35, 1903, p. 87; Hausdorff, F., in *Leipziger Berichte math. phys. Klasse* , 59, 1907, p. 217.

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crowds, having such orders equal to the first number of the third class of ordinal transformation is finished.

It is customary succession orderly sequence consisting of ordinary integers. Each of these sequences can be associated with the two-way one binary fraction, in the range between 0 and 1; for example, the sequence of 1, 3, 5,...,ii 0.101110111110 ... is associated binary fraction. The first principle generator acts to raise by one unit value of each number in a given sequence. The second principle generator acts upon a succession of succession applied its cantorian a diagonal process. The entire building begins with a sequence of units. Presented below O. Becker's example:

1	1111 ...
2	2222 ...
3	3333 ...
.	....
ω	1234 ...
ω + 1	2345 ...
ω + 2	3456 ...
.	....

ω ω + 1357

The problem, besides the construction of trans ordinals - finished off the left edge of the scheme is to *determine* a grading *constructive* ordinals trans - finished the second class of numbers matter more precisely delineated by A. Church and SC Kleene, who also set the following requirements:

1. Each ordinal appears on the way to be denoted by one symbol (the first is 1);

2. If there are two ordinal denoted by two different symbols, to be able to decide using a recursive process arithmetic of the two is more and less;

3. Given a random order, different from 1 to be able to determine whether it has an immediate predecessor or not;

4. Given a certain order is required to be able to demonstrate the constructive way that it can reach there by induction of trans - finite.

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These requirements show that both SC Kleene A. Church and its design are on the field as interpreted by O. Becker[27]. This author believes the fact that *ordinals trans-finished* a role "constructive" for mathematics in general and specifically for research relating to its fundamentals, they found their applications in characterizing the exact degree of complication of the expressions logical - mathematical, the schemes deductive and constructive processes.

This serves in particular to performing demonstrations consistency. O. Becker who cites G. Gentzen, who failed to show consistency first complete theory of numbers[28]; he could prove the finite nature of arithmetic, because it applied a *reduction* process represented in a form logistics. To obtain this demonstration finiteness of arithmetic, arithmetic building he associated each a serial number, ensuring that after each step of the reduction, the ordinal to shrink. O. Becker concludes that while such a constructive process exceeds the infinite complexity of other constructions (deductive chains) are required ordinal trans - finished their entirety can be conceived only about using a inductions inductive trans - finite"[29].

O. Becker believes that *restricting the concept of constructability math* or even countable enumerable sets, we can see the presence of a limitation in developing mathematical concepts. This limit "imposed by mathematical thinking it themselves", says O. Becker.

In fact, he devotes an entire chapter to the issue limits mathematical thinking, speaking *immanent limits of mathematics* in entering and above the *limit philosophical thinking* and *mathematical problem,* over which a longer return.

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