ABOUT AN INTERACTIVE METHOD OF STOCHASTIC PROGRAMMING

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Abstract. This paper deals with a model of stochastic programming named distribution problem. The problem in this case is formulated and the paper presents a way of solving it by an interactive numerical method.

Keyword: Stochastic programming.

1. Introduction

It is known that if at least one of the coefficients of the matrices A, b and c of a linear programming problem are random variables, then we obtain a *stochastic programming problem*.

Considering the triplet (A, b, c): $\Omega \to R^{mn+m+n}$ and assuming that for each $\omega \in \Omega$ there can be solved in a certain way the deterministic linear programming problem

$$\min (\max)\ z = c'(\omega)x$$

subject to:

$$A(\omega)x \le b(\omega)$$

$$x \ge 0$$

this means that optimal decision making can take place after the realization of coefficients.

Considering these aspects, different approaches on solving stochastic programming problems have been developed. One of this approach is based on the "wait and see" principle, known as *the distribution problem*; in this case it is pursued that before finding the optimum value we know some data about the z (ω) or the optimal decision x (ω), as for example random variable distribution or certain characteristics (mean, variance, etc.). From the viewpoint of the decision maker this approach has a passive character and does not lead to a "a priori" definiton of a decision.

I present below some theoretical concepts on the problem of distribution and then the application of a numerical method for solving it proposed by the author.

2. The distribution problem and the interactive method for solving it

Let be A(ω), b(ω), c(ω) the random matrices with given dimensions in (LP), having random variable elements defined on the field of probability {Ω, K, P}. For each $\omega \in \Omega$ we associate with the triplet A(ω), b(ω), c(ω) the stochastic linear programming problem:

$$(LS_{\omega}) \qquad \max_{x \in D(\omega)} (\min) z(\omega) = c'(\omega)x$$

$$D(\omega) = \{x \in R^{n} \mid A(\omega)x \le b(\omega),\ x \ge 0\},\ \omega \in \Omega$$

The values $\max_{x \in D(\omega)} (\min) c'(\omega)x$ *are not known before knowing the exact values of the* *coefficients and we want to find the distribution function and (or) the mean or variance of z (ω) or (and) the probability that the optimal value to be found out within a certain range.*

The determination of the distribution function was made by many mathematicians since 1955 by G. Tinter in [7], or by Gh. Mihoc who introduced in the same year a way of solving the problem of the optimum distribution.

B. Bereanu has shown in [3] how to determine the exact value of the distribution function of the optimal value z(ω), using the mathematical programming method and the notion of "decision region" in stochastic linear programming. The same author uses the Laplace transform to determine the distribution function in an article published in 1966 [4]. Among the approximate methods used by Bereanu to determine the distribution function of z (ω) the cartesian integration method is to be mentioned.

There are various numerical methods for solving problems of distribution (determining the distribution function or its main characteristics) that produce a desired approximation of the values required. The number of calculations increases with the desired accuracy.

We mention here one of these methods, called *simulation method* and present an algorithm for this method proposed by the author.

The simulation method requires the use of computer and consists in generating a set of triplets (A_n, b_n, c_n) from random population (A(ω), b(ω), c(ω)), where the sequence elements are independent, so that (A_n, b_n, c_n) sufficiently approximate (A(ω), b(ω), c(ω)).Therefore we consider the problem (LS_ω) for which we assume that there is a possible solution, z(ω) is finite with probability 1 and the mean value exists. We calculate the value of the function z(ω) for a given N:

$$z_N(\omega) = \frac{1}{N}\sum_{n=1}^{N} z(A_n, b_n, c_n) \qquad (2.1)$$

Based on the law of large numbers there holds for any $\varepsilon \geq 0$

$$\lim_{N\to\infty} p(\omega \,|\, z_N(\omega) - \mathbf{E}(z(\omega))| \geq \varepsilon) = 0 \qquad (2.2)$$

Through the simulation method there may be obtained:

a) *variance* $\mathbf{D}^2$ (z(ω)) in this case the problem is solved by methods being specific for the two stages : the first sample is used to estimate the variance of z (ω) and the more general sample to estimate the mean value of z(ω).

b) *distribution function.* For this we define a random variable γ(y) such that:

$$\gamma(y) = \begin{cases} 1, & z \leq y \\ 0, & \text{altfel} \end{cases} \qquad (2.3)$$

and we approximate (2.3) through $\frac{1}{N}\sum_{n=1}^{N}\gamma_n(y)$.

One of the directions of approach the simulation method is the one seeking the global search of the stochastic optimum.

The arising question is to search the elements of a finite set of alternatives of a configured system that optimizes the system performances given that they can not be evaluated analytically, but they can be evaluated by simulation. In this type of problems extracting samples from a finite number of alternative is of interest.

A technique that we used to develop the above mentioned interactive method was proposed by Sigrun Andradóttir în [1], which, unlike other existing techniques, used only the observing of the objective function for each value of the parameter at each iteration, and it was resumed with some differences in [2]. What does this method solve? Each iteration finds a best global optimum. It is assumed that any two points are neighbors: this ensures that the solution is compared to all other alternatives.

Let D be a finite set of different alternatives, D = {1, 2, ..., N}. Let be optimization problem

$$\min_{n\in D} f(n) = \{x_n\} \qquad (2.4)$$

where $\{x_n\}$ is a sequence of random variables; we consider $S\subset D$ set of global minimums of the function f.

When the mean $\mathbf{E}\{x_n\}$, n =1,..., N can be evaluated analytically, the optimization of problem (2.4) can be solved using the integer programming technique, otherwise there can be applied direct simulation. The presented method aims at locating an element of S in cases where the mean $\mathbf{E}\{x_n\}$, n = 1,..., N can not be evaluated analytically. We make the following assumption :

For every i, j $\in$ D there exists the random variable $y^{(i\to j)}$ such that for every i $\in S$, j$\in S$ and n$\in$D, n $\neq$ i, j.

(1) $P\{y^{(j\to i)} > 0\} > P\{y^{(i\to j)} > 0\}$

(2) $P\{y^{(n\to i)} > 0\} \geq P\{y^{(n\to j)} > 0\}$

(3) $P\{y^{(i\to n)} \leq 0\} \geq P\{y^{(j\to n)} \leq 0\}$

In [9] it has been shown that this assumption holds for all i, j $\in$D and we consider in this case $y^{(i\to j)} = x_i - x_j, \forall$ i,j $\in$ D.

The variables $y^{(i\to j)}$ will be used to compare the states *i* and *j*. If in an iteration we decide to compare the current point *i* with the point alternative *j* , it means that we run an experiment or a simulation to obtain the random variable $y^{(i\to j)}$. The rule of passing to the next point is the following: if $y^{(i\to j)} \leq 0$ then *i* will be the next point, otherwise that will be *j*. If we assume that i $\in$S and j$\notin$S then the inequality (1) expresses that the probability of moving from *j* to *i* (and then we take the right decision) is greater than the probability of moving from *i* to *j* (and then we take a incorrect decision). The inequality (2) states that the probability of moving from the current point *n* is higher for point *i* than to point *j*.
The inequality (3) states that the probability to stay in the current point *i* is higher than the current point j when it comes to move towards a point n $\neq$ i, j.

Starting from the previously mentioned algorithm, I tried to improve the way of determining the optimal solution by increasing the speed of finding it; how can this be done? In the presented algorithm we can observe that, if one point is chosen randomly, each time the algorithm is restarted from step 1 to 3. The author proposes in its algorithm that each point *n*' of the set D has attached to a counter that records the level function at that point.

If there is found one single point *n* '' where the function value is lower, the point n' is to be removed from the subsequent requests.

We also insert a counter for counting the achieved points in such a way that it forces stopping the algorithm if all points have been checked; below I present the notations I used and the working algorithm.

m - iteration number, where m=1,2,...

k_m - the current point for all n□□D

k(n) -counter counting stationary point n . If the system was in state n k(n) takes the value 1 otherwise it takes the value 0.

N(n) - counter level counting of function in state n If there are more convenient states in relation to it, it takes the value 1 otherwise it takes the value 0

X_n – objective function value at point n

s – counter for counting the achieved points

The algorithm is as follows:

step 0: Enter the number of alternatives N, choose a state k_0=n, n□□D={1,…,N}
as a starting point, determines Xk_0. Let m = 0, N(n) = 0 be for every n□D and
let be k(n) = 1 for n = k_0 else k(n) = 0.

step 1: Generate a variable V_m such that for all n □D, n□□□k_m to have
V_m = n with probability 1/ N-1.

step 2: If k(V_m) = 0 then s=s+1, determine XV_m and go to step 3 otherwise go to step 4.

step 3: Xkm – XVm □□□□□then N(k_m)=1,m = m+1, k_m=V_{m-1}, k(V_{m-1})=1, Xk_m=
XV_{m-1} and go to step 1

step 4: If s = n stop otherwise go to step 1.

The algorithm enables solving a distribution problem for different cases. It remains to analyze the behavior of the algorithm in terms of number of iterations, for example the case of a Poisson distribution with parameter λ and different values of n, and comparing the rate of convergence with that of other methods.

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