APPLIED MATHEMATICS, INFORMATION TECHNOLOGY AND STATISTICS

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SINGULAR VALUE DECOMPOSITION IN AHP
S. I. GASS, T. RAPCSÁK

FACING GLOBALIZATION
POÓK László, Liviu CIUCAN-RUSU

MATHEMATIC MODEL FOR NATURAL SCIENCE AND ECONOMY
Horea POP

TITLE WILL BE COMMUNICATE LATER
Osei ADJEI

EXISTENCE THEOREMS FOR ABSTRACT EQUATIONS AND APPLICATIONS
Silviu SBURLAN

FORMAL METHODS IN SOFTWARE ENGINEERING EDUCATION
Militon FRENȚIU

LES ABAQUES DE CALCUL ET LEURS APPLICATIONS À LA GESTION D'ENTREPRISE
Georges BALAYE, Liviu CIUCAN-RUSU

CONSTRUCTING EULERIAN AND HAMILTONIAN PATHS IN GRAPHS
KÖRTESI Péter

AN APPLICATION OF GROEBNER BASES
HORVÁTH Alexandru

CHARACTERIZATION OF BUSEMANN HYPERBOLICITY ON BERWALD MANIFOLDS
KRISTÁLY Alexandru, KOZMA László, RÓTH Ágoston

KNOWLEDGE REPRESENTATION METHODS IN EXSYS DEVELOPER
Laura-Diana GENETE, Ionela BÂCÂIN

MOBILE COMMERCE: A NEW WAY OF BUYING AND SELLING
Daniela RUSU

THE (IN) SECURITY OF MOBILE COMMERCE
Daniela RUSU

MATHEMATIC VERSUS BIOLOGIC IN NEURAL NETWORKS
Călin ENĂCHESCU

A SIMPLIFIED QUICK-HULL ALGORITHM FOR THREE/DIMENSIONAL SPACE
Ion COZAC

UNIDIMENSIONAL ANALYSIS MODEL (ANOVA)
Daniela DĂNCIULESCU, Daniel DĂNCIULESCU,

PARTICULAR TYPES OF MODELS FOR TIME SERIES ANALYSIS AND ANALYSIS OF PANEL DATA
Ana-Gabriela BABUCEA

SOME COMPUTATIONAL ASPECTS OF THE SYMMETRIC EIGENPROBLEM
SZABÓ Zsuzsanna

MEDICAL STATISTIC ANALYSIS SYSTEM
Daniela DĂNCIULESCU, Virgil POPESCU

PROBLEMS OF DECISION ON INDETERMINACY CONDITIONS
Tomiţă VASILE, Cora Ionela DĂNIASA

MATHEMATICAL MODELING IN PROBLEMS OF DECISION
Tomiţă VASILE, Dragoş STUPARU

INDUSTRIAL DATA ACQUISITION CENTRALIZATION AND DISPLAY
Traian TURC, Gavril TODEREAN

HOW A SME CAN SPIN-OFF FROM A UNIVERSITY WITH SUPPORT OF EU-PROJECTS
Dimitris KARAGIANNIS, Elena Teodora MIRON

COMPONENT BASED APPROACH FOR SCIENTIFIC VISUALIZATION OF EXPERIMENTAL DATA
Dumitru RĂDOIU, Adrian ROMAN

STATISTICAL METHODS OF GROUPING THE TERITORIAL ADMINISTRATIVE UNITS OF MURES DISTRICT BASED ON THE COLLECTED INCOMES
Daniela ŞTEFĂNESCU, Gabriela VAIDA

MANAGEMENT SISTEMS FOR INFORMATION SECURITY
Călin SARCHIZ

SECURE E-BANKING
Călin COMES

AML FOR FINANCIAL REPORTING
Călin COMES

MULTIPLEXED SYSTEMS FOR SERIAL DATA LINES
Traian TURC, Gavril TODEREAN

DISTRIBUTION TRANSPARENCY
Ileana ŞTEFAN

FRAGMENTATION IN DISTRIBUTED DATABASE SYSTEMS
Ileana ŞTEFAN

SINGULAR VALUE DECOMPOSITION IN AHP

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The Analytic Hierarchy Process (AHP) (Saaty, 1990) has been accepted as a leading multiattribute decision model both by practitioners and academics. AHP can solve decision problems in various fields by the prioritization of alternatives. The heart of the most familiar version of the AHP is the Saaty's eigenvector method (EM) which approximates an $n \times n$ positive reciprocal matrix $A = (a_{ij})$, $(a_{ij}) = \dfrac{1}{a_{ij}}$, $i, j = 1, \ldots, n,$, by a vector $\mathbf{w} \in R_+^n$, where R_+^n is the positive orthant of the n-dimensional Euclidean space R^n, such that the matrix of the ratios (w_i / w_j), $i, j = 1, \ldots, n,$ is the best approximation to A, in some sense. It is emphasized that the EM results in a priority vector $\mathbf{w} \in R_+^n$ and an inconsistency number $_{\max}$.

However, the EM has been criticised both from prioritization and consistency points of view and several new techniques have been developed.There are two different approaches in the AHP: deterministic and statistical or stochastic.

In the deterministic approach, the underlying assumption is that one can observe the preferences with certainty and the only uncertainty lies in the elicitation of these preferences which give rise to the inconsistency condition. The EM is incorporated into the deterministic approach. Other approaches include the least-squares method (LSM) (Jensen, 1984), the chi-square method (CSM), (Jensen, 1984), the logarithmic least-squares method (LLSM) (Saaty and Vargas, 1984) and the weighted least-squares method (WLSM) (Chu et al., 1979). In the statistical approach, it is

assumed that the preference judgments are random variables associated to an unknown probability distribution.

In [12], Saaty compares EM with LLSM and states that EM captures the inconsistency of dominance, while LLSM minimizes a distance function and searches for symmetry without an explicit attempt to capture dominance. But inconsistent dominance judgments are asymmetric. In this case, EM and LLSM give rise to different derived scale, and, at times, to a different choice. Moreover, he summarized ten reasons for not using LLSM instead of EM.

In [1], WLSM is compared with EM. WLSM has the advantage that it involves the solution of a system of linear algebraic equations and is, thus, conceptually easier to be understood than EM. Comparisons show that sums for WLSM are less than those for EM, and the dominant weight seems to be larger for WLSM.

Throughout Saaty's work only the right eigenvectors are used. However, theoretically, the use of left eigenvectors should be equally justified. This problem was observed first by Johnson et al. (1979). In [2], a new method, known as the Modified AHP (MAHP), claimed that the right and left eigenvectors inconsistency problem can be effectively reduced. In [14], an attempt was made to compare AHP and MAHP by using 42 models comprising 294 reciprocal matrices. It was revealed that MAHP is not better than the AHP.

Singular value decomposition (SVD) is an important tool of matrix algebra that has been applied to a number of areas, for example, principal component analysis and canonical correlation in statistics, the determination of Moore-Penrose generalized inverse, and low rank approximation of matrices, Kennedy and Gentle (1980), Eckart and Young (1936), Greenacre (1984). The matrix algebra and computational aspects of SVD are discussed in Kennedy and Gentle (1980), and statistical applications are described in Greenacre (1984). The aim of this lecture is to show that the SVD seems to be a good tool to provide a theoretically well-founded solution in the AHP both for the scaling and consistency problems and to develop further the results of Gass and Rapcsák (1998), as well as Standard (2000) in order to derive the weight vector in a convenient and consistent way in the AHP. The SVD approach is illustrated and compared to the EM for some numerical examples.

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FACING GLOBALIZATION

(A research report on data collected in Romania, Hungary and South Africa)

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Abstract: *Responses of university students from Romania, South Africa and Hungary were surveyed on a 24-point questionnaire on issues of opening national markets, globalization, and joining regional economic unions. In an environment of increasing competitiveness respondents were asked to decide on the nature of information they were most likely to use for business decision-making. The history, background and cultures of the three countries were as different as their responses to how they were expecting to face globalization. While some expectations were similar political, cultural and economic backgrounds were likely to have been the sources of differences. Statistical analysis was used to examine survey data using analysis of variance and correlations among variables.*

1. Introduction

Globalization has the ability to alter national cultures, influence flows of capital and labor, and change traditional ways of doing business. Globalization is driven by communications and information technology. Opening national markets to globalization is often national policy as is the building and financing of the infrastructures needed to support it. However, once national markets have been opened to international competition these markets "gain greater access to more capital, technology, cheaper imports and larger export markets" (African Development Bank, 2003, p. 160), as well as to strategic market information.

The extent to which a nation can benefit from the forces of globalization depends on the ability of its population to see these forces as working to their benefit, instead of against them. Globalization brings with it opportunities to harness new technologies which, in some instances

may run counter to popular cultures and national ethical values. Can the nation alter its value system or even except the change? Or, will they resist the change until the benefits pass by them?

Globalization has been associated with negative developments as well, e.g. environmental degradation, exporting of jobs, minimizing self-determination, diminishing authority of the state, and further abandonment of the poor. Clearly, the ability to successfully compete in an open market depends on a firm's ability to muster human, financial, informational and material resources as well as have the basic support of its own national government. In this paper the authors will examine popular views of and responses to the forces of globalization as expressed by students of three developing countries: Hungary, Romania, and South Africa. We will focus particularly on the reactions to globalization of university students because they will be the decision makers and beneficiaries, if there are such, of this newest wave of globalization sweeping much of the world.

What is globalization? It is the opening of national markets to international competition, it is the harmonization of the legal system with our competitive partners, and it is the use of knowledge, technology and information in order to gain competitive advantage.

2. The study

Country backgrounds

All three countries have suffered under oppressive regimes for nearly five decades and received their freedoms within the past fifteen years. Hungary experienced slow, deliberate changes since about 1965 from a centrally planned economy toward liberalization of business processes through cautious economic reforms. Hungary is slated to join the European Union in 2004. Romania experienced a brutal communist dictatorship until 1989, when a violent revolution resulted in repeated attempts at the establishment of democratic institutions. More recently Romania has been aspiring to become a member of the EU as well. From the late 40's until 1994 South Africa has shown how effectively a distorted but legalized distribution of power can bring about a warped social system, when backed by strong-willed security forces. Since the banishment of apartheid South Africa too has embarked toward a democratic government and toward economic integration with the rest of the world.

While recent history allows us to draw parallels between the three countries many social, political and economic differences distinguish them. South Africa is a diverse multi-racial multi-cultural country with a sad background of slavery and minority subjugation. Romania is a patchwork of smaller countries acquired through agreements and international edicts. Nevertheless Romania's population is mostly white of Latin origins with a small minority of

gypsies. Hungary today is the remnant of an empire populated by a mostly monolithic group of the Magyars alternatively deemed either of oriental descent or white. These social differences are observable in the contrasts of the cultures. Prior to 1994 democracy has been the privilege of the South African elite, Hungary [another expression like tentatively, partially??] embraced it at the turn of the last century, while Romania always struggled with it, hopefully finally getting it right by the late 90's.

Political and social history is reflected in these countries' economic performance. Table 1 below depicts GDP per capita over nearly two decades for all three countries. This reinforces the observation that Hungary's economy was on the upswing even towards the end of communist rule in 1989 (often attributed to "goulash communism"), while South Africa was suffering the ravages of social upheavals well into the end of the 20th century. Though, shortly after the fall of apartheid South Africa's markets began opening up. Romania, due to a lack of democratic history had difficulties embracing free market capitalism even after liberation in 1989. Not until about 2000 did Romania begin instituting open market policies as reflected in Table 1 below. GDP/capita in the US is included for comparison.

One needs to differentiate, however, between export orientation and globalization. One produces goods for sales externally, while the other reorients its economy to the exchange of goods and services on a world market.

Description of the study and data collection

The questionnaires targeted upper and lower division university students in South Africa, Romania, and Hungary. Table 2 below offers descriptive statistics of the university students queried in Romania. For the sake of brevity South African and Hungarian descriptive statistics are omitted.

The questionnaires were administered in the native languages of the students except in South Africa where of the eighteen languages spoken English as one of the official languages was chosen because of its wide spread use. Questions were examined by local individuals to remove potential bias. Of the 18 questions the first five collected demographic information. The remainder of the questions addressed information on the respondents' understanding of information and information system needs in a globalized world, responses to international completion available to domestic firms, technological tools available for business survival, the respondents' opinions on their countries' preparedness for a

Table 1 History of GDP/Capita of the three countries with a comparison to USA data

Country	Years					
	1980	1985	1990	1995	2000	2002
Hungary	4199	4637	4857	4366	5425	7277
Romania	1775	2022	1702	1564	1460	1987
South Africa	4620	4229	4113	3863	3985	3392
USA	21001	23384	26141	27713	31996	36123

Data: World Bank, 2004; CIA World Fact Book, 2004

competitive world market, for globalization, and their ability to tolerate risk. This paper focuses on the information aspects of the study, which is addressed by the first three issues above: information, systems, technology as a competitive weapon, and utility and ethics of use of technology tools.

Three questions were used to solicit responses to the issue of "information use and use of information systems". These are, 1. "Should the information that you will use in your future employment describe", (with possible selections from narrow to broad response); 2. "Should the information system that you will use at work provide you with information describing", (with possible selections of answers from narrow to broad response); and, 3. "In general, should organizational information systems provide employees with information describing", (with possible selections from narrow to broad response).

Two questions were used to solicit responses to the issue of "foreign competition and domestic responses". These are, 1. "Foreign firms can easily out-compete our businesses because foreign firms are funded (and/or capitalized) at higher levels than we are and therefore they can execute a more thorough marketing program", (with possible selections from agreement to disagreement); 2. "There are technological methods available to small domestic businesses with which they can out-compete well funded foreign businesses", (with possible selections from agreement to disagreement).

Four questions were used to collect responses to the issue of firms having "technological tools vs. their utility and the ethics of using them". These are, 1. "Domestic firms might be able to use electronic commerce sites to collect valuable data about customers and their preferences", (with possible selections from agreement to disagreement).

Table 2 Descriptive statistics of the data used

Information use and use of IS	Country	N	Mean	Std. Dev.
Should the information that you will use in your future employment describe	Hungary	515	3.07	0.91
	S Africa	651	3.61	0.72
	Romania	367	2.95	1.01
Should the information system that you will use at work provide you with information describing	Hungary	515	3.17	0.86
	S Africa	648	3.49	0.81
	Romania	368	3.05	0.99
In general, should organizational information systems provide employees with information describing	Hungary	515	3.01	0.99
	S Africa	647	3.45	0.81
	Romania	367	3.07	1.02
Technology as competitive weapon	**Country**	**N**	**Mean**	**Std. Dev.**
Foreign firms can easily out-compete our businesses because foreign firms are funded (and/or capitalized) at higher levels than we are and therefore they can execute a more thorough marketing program	Hungary	516	1.46	0.81
	S Africa	646	1.85	0.85
	Romania	368	1.38	0.68
There are technological methods available to small domestic businesses with which they can out-compete well funded foreign businesses	Hungary	516	1.56	0.82
	S Africa	645	1.96	0.83
	Romania	368	1.68	0.80
Technological tools vs. their utility and ethics of use	**Country**	**N**	**Mean**	**Std. Dev.**
Domestic firms might be able to use electronic commerce sites to collect valuable data about customers and their preferences	Hungary	516	1.30	0.66
	S Africa	647	1.14	0.45
	Romania	368	1.18	0.50
Customer tracking data collected on the Internet can be used to make strategic business decisions if made available to decision makers	Hungary	516	1.59	0.87
	S Africa	644	1.19	0.49
	Romania	367	1.30	0.62
Collection of customer data on electronic commerce sites for the benefit of the firm is ethically acceptable	Hungary	516	1.93	0.93
	S Africa	644	1.68	0.82
	Romania	367	1.64	0.81
Data collected on the Internet is more reliable than data collected via observation and manual recording	Hungary	516	2.60	0.73
	S Africa	643	2.35	0.82
	Romania	368	2.36	0.81

2. "Customer tracking data collected on the Internet can be used to make strategic business decisions if made available to decision makers", (with possible selections from agreement to disagreement). 3. "Collection of customer data on electronic commerce sites for the benefit of the firm is ethically acceptable", (with possible selections from agreement to disagreement), 4. "I have more confidence in data collected on the Internet than in data collected via observation and manual recording", (with possible selections from agreement to disagreement).

Results

Results of multiple comparison tests using one-way ANOVA are in Table 3 below. It should be noted that the three countries responses on the nine questions are statistically significantly different, on the 0.01 level, with sufficiently high F-values.

Addressing the first of the three issues posed above: **information use and the use of information systems**, South African university students need significantly (<0.05) broader based information for business decision making than Hungarians or Romanians do, and they are expecting this broader information to be provided to them by the organizational information system, with a significance of 0.05. Furthermore, South Africans expect that their peers at the organization would receive broad based information as well, more so than Hungarians and Romanians do, with a significance of 0.05. These expectations for broader based information and systems may be indicative of South African students' more advanced global vision. No significant differences were found between Romanian and Hungarian students' need for business information in their future employment.

Table 3   Multiple comparison tests for the three country data

	Between countries			Mean differences		
Information use and use of IS	**F**	**Sig.**	**Country**	**Hungary**	**S Africa**	**Romania**
Should the information that you will use in your future employment describe	89.71	0.000	**Hungary**	n/a	-0.54*	0.12
			S Africa		n/a	0.66*
			Romania			n/a
Should the information system that you will use at work provide you with information describing	35.15	0.000	**Hungary**	n/a	-0.32*	0.11
			S Africa		n/a	0.43*
			Romania			n/a
In general, should organizational information systems provide employees with information describing	37.67	0.000	**Hungary**	n/a	-0.44*	-.06
			S Africa		n/a	0.38*
			Romania			n/a
Technology as competitive weapon	**F**	**Sig.**	**Country**	**Hungary**	**S Africa**	**Romania**
Foreign firms can easily out-compete our businesses because foreign firms are funded (and/or capitalized) at higher levels than we are and therefore they can execute a more thorough marketing program	52.38	0.000	**Hungary**	n/a	-0.39*	0.08
			S Africa		n/a	0.46*
			Romania			n/a
There are technological methods available to small domestic businesses with which they can out-compete well funded foreign businesses	36.91	0.000	**Hungary**	n/a	-0.40*	-0.11
			S Africa		n/a	0.29*
			Romania			n/a
Technological tools vs. their utility and ethics of use	**F**	**Sig.**	**Country**	**Hungary**	**S Africa**	**Romania**
Domestic firms might be able to use electronic commerce sites to collect valuable data about customers and their preferences	14.09	0.000	**Hungary**	n/a	0.17*	0.12*
			S Africa		n/a	-0.05
			Romania			n/a
Customer tracking data collected on the Internet can be used to make strategic business decisions if made available to decision makers	53.42	0.000	**Hungary**	n/a	0.40*	0.30*
			S Africa		n/a	-0.11*
			Romania			n/a
Collection of customer data on electronic commerce sites for the benefit of the firm is ethically acceptable	16.93	0.000	**Hungary**	n/a	0.25*	0.29*
			S Africa		n/a	0.04
			Romania			n/a
Data collected on the Internet is more reliable than data collected via observation and manual recording	17.27	0.000	**Hungary**	n/a	0.25*	0.24*
			S Africa		n/a	-0.01
			Romania			n/a

*The mean difference is significant at the .05 level.

On the subject of foreign **competition and domestic responses** the data shows that South African students are statistically more concerned, with a significance of 0.05, about foreign firms having the ability to out-compete domestic businesses due to foreign firms having the economic muscle and know-how. On the other hand, South African students have less faith in the ability of technology at the disposal of smaller domestic firms to out-compete foreign firms, with a significance of 0.05, than Romanians and Hungarians. However, in absolute terms, SA students are split on both of these issues. These latter countries' university students are statistically similar in their thinking about foreign firms' competitive abilities, and believe that their domestic firms can rely on technology to out compete foreign firms.

When examining the issue of the **utility of specific technologies such as the Internet, and the ethics of using such technological tools**, Hungarian students indicated that their domestic firms would not find the to use of electronic commerce sites to collect data about customers and their preferences valuable, should not use such tracking data collected on the Internet to make strategic business decisions, find such data unethical to use, and data collected via the Internet is less reliable than data collected manually, significantly more so (<0.05) than South Africans or Romanians. Collectively, however, all three countries' university students are in favor of collecting customer data on the Internet, using it for strategic decision making, find such activity more ethical than not so, and all three tend toward believing that such data is less reliable. Romanians were found to see the value in using tracking data collected on the Internet to make strategic business decisions significantly more so than South Africans students (<0.05); there were no statistical differences noted between Romanians and South Africans regarding the utility of e-commerce sites, the ethics of collecting customer data on the Internet, or regarding the reliability of data collected on the Internet.

Interpretation of the data

Overall, university student responses to the globalization of industries in their respective countries were significantly different. Regarding the comparatively more advanced vision of South African students toward information and information systems in business decision making credit may be due to specific and targeted university programs in the field of business information systems as compared to technologically oriented programs in Romania and Hungary. Nevertheless seeing information and information systems as necessary ingredients of business decision making in a globalized world South Africans have a step up on their Hungarian and Romanian counterparts.

Relative to Romanians and Hungarians, South African students view foreign firms as more of a threat to their domestic industries and not view the power of technology as a tool for equalizing

competition, is again more likely the result of a broader based, multidisciplinary university curriculum allowing South Africans to have a more integrated and critical view of world business. In absolute terms, however, South Africans were divided both on the issue of external threat and on the equalizing ability of technology. Hungarian and Romanian universities adhere to a traditionally hierarchical academic structure where humanistic and technical subjects are mostly designated to be at separate institutions. South Africans on the other hand, tend to adhere more to a western model of academia where most subject areas are within the same institution and students receive a broader training, though possibly lesser in depth.

The Hungarian skepticism of the technical superiority of the Internet at collecting, and providing unerring data is fortuitous to the extent that it tempers overconfidence in new technology. It may also be based on wide spread national distrust of all things new and untested, and to some extent distrust of higher authority. On the face of it this may be healthy skepticism at embracing new technology before trusting it. Romanians and South Africans, relatively speaking, may be disregarding the ethical issues of privacy, fraud, potential for political and criminal surveillance, as compared to Hungarians, just because the apparent anonymity of the Internet and its technological elegance hide such transgressions. Comparatively, Hungarians do not appear to have such faith in the Internet and are less likely to trust it. In absolute terms, all three nations are tending toward distrust of Internet based data. Romanians are more likely to use customer-tracking data than South Africans would. One cannot help but wonder if the persistence of oppressive political systems both in Romania and South Africa well beyond that in Hungary may have conditioned these nations to be less cautious of intrusive Internet technologies.

At the same time the social, humanistic, legal, and ethical aspects of the Internet may be manifest in the broader education system prevailing in the South African academic model than those in Hungary and Romania.

3. Conclusions

This article is reporting on results of a comparative study by comparing statistically the responses of university students on issues of globalization and information systems in three countries. No criticism is intended or implied. We did not apply any standards; we were only making observations based on differences or similarities.

Preliminary results in the study indicate that South African students' background may have better prepared them for the competitive business world heralded by globalization than observable among Hungarian and Romanian students. They also have a relatively better understanding of the relationship between competitiveness and the use of information technology. In order to cope and

even excel in today's global business and economic environment enterprises need to transform themselves to be more flexible and short term reactive. The use of information and technology is key to becoming so.

The integrated nature of South African university curriculum may be part of the reason for their students' broader views and attitudes. As such there may be a suggestion here for Hungarian and Romanian institutions to more purposefully integrate business and technical education in the curriculum, more specifically business information systems. Breadth of vision and the ability to look beyond the elegance of technical solutions may be germane to economic survival in a globalized world economy. Furthermore, emerging nations need to throw off the vestiges of failed empires, distorted social, economic and political systems and embrace the reward based global market economy, even if it appears less certain at times.

LES ABAQUES DE CALCUL ET LEURS APPLICATIONS A LA GESTION D'ENTREPRISE

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Résumé: *Notre demarche se propose de presenter un outil statistique pret à etre utilisé dans les decision concernant la gestion de l'entreprise. L'utilisation des echelles logarithmiques nous permet de construir des graphiques tres facile à utiliser, un soutien statistique à la portée d'un manager. A ce propos, en transformant les courbes en droites on construit les abaques, véritables tableux de bord pour etudier plusieurs phénomènes qui se passent dans la vie de l'entreprise. Soie qu'ils sont de l'addition, de la soustraction où bièn de la multiplication et de la division, les abaques sous leur presentation graphiques nous ont offert l'opportunité de trouver quelques applications pratiques dans la statistique des affaires.*

On commence par presenter les avantages apportés par l'utilisation des echelles logaritmiques. Tout d'abord on presente dans la Figure #1 un graphique de coordonnées à échelles logarithmiques.

Figure #1
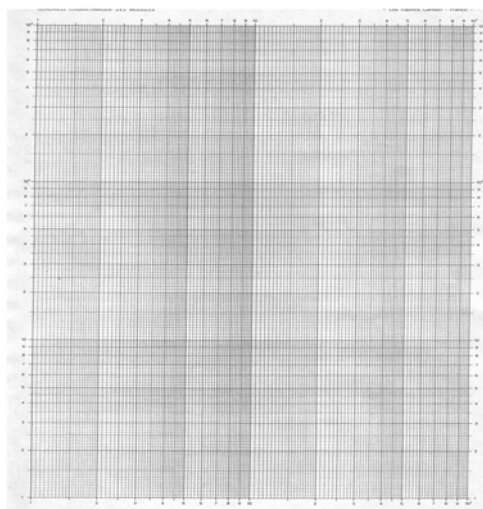
Si nous comparons un graphique logarithmique à un graphique à échelles millimétrées ordinaire, nous constatons les différences suivantes:

1. La première constatation qui s'impose, est que dans un graphique logarithmique, les distances entre les graduations sont inégales. La distance entre 1 et 2 est supérieure à la distance entre 2 et 3, elle-même supérieure à la distance entre 3 et 4, etc.. et vont se resserrant jusqu'à 10. Ceci est dû à ce qu'en logarithmes, les distances représentent des *valeurs proportionnelles* et non des valeurs brutes comme dans les graphiques à échelles atithmétiques. Dans les graphiques millimétrés, les échelles représentent des distances égales et des proportions inégales. Dans les graphiques logarithmiques, les échelles représentent des distances inégales et des proportions égales.

De là découlent deux conséquences pratiques importantes :

Si nous parlons des graphiques de contrôle, comment obtenir des plages de +/– 5%, +/-10%, etc.. de part et d'autre de la bissectrice. Ces plages se répartissent en éventail de part et d'autre de la bissectrice (Figure #2).   **Figure #2**
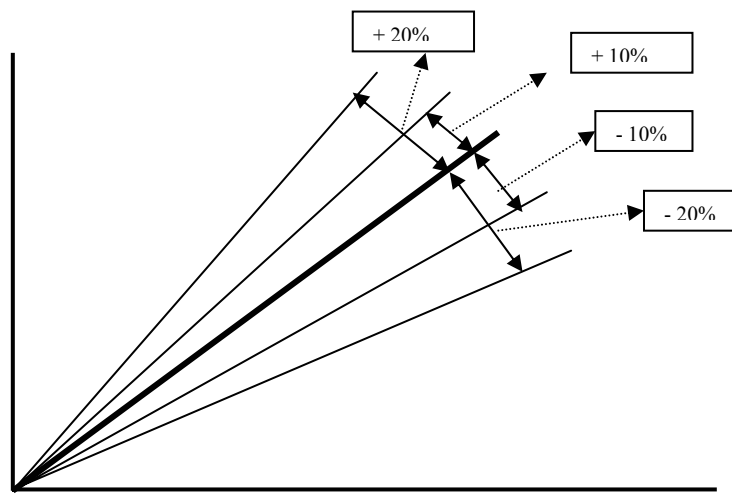
Ce même graphique en coordonnées logarithmiques, se présente ainsi (Figure #3) :

Figure #3
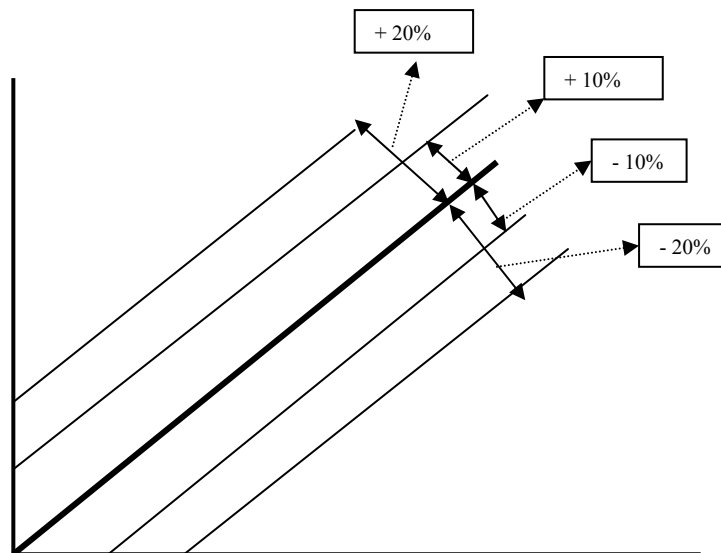
2. L'origine des échelles est 1 et non pas zéro comme dans les graphiques ordinaires, puisque le logarithme de zéro n'existe pas. La distance entre la graduatio 1 et 10 est un *module.* Le graphique IV.3.1. fait partie du type 3x2 modules, c'est à dire qu'il comprend 3 modules à l'échelle verticale et deux modules à l'échelle horizontale. Tous les modules sont égaux entre eux : Entre 1 et 10, 10 et 100, 100 et 1.000, on observe la même distance.

3. Les graduations sur les quadrillés logarithmiques, sont préétablies. On pourrait penser que cette particularité pose un problème par rapport aux quadrillés arithmétiques qui nous laissent la possibilité de disposer nos échelles comme nous voulons. En réalité, ces gradu- ations préétablies présentent un avantage certain, c'est de nous dispenser d'avoir à calculer nos échelles : elles sont déjà disponibles quelles que soient les données à enregistrer. Notre seule intervention consiste à en définir l'origine : soit nous la laissons telle quelle : 1 pour le premier module, l'origine du deuxième module est alors 10 et celle du troisième 100. Ou bien nous décidons d'établir comme origine 10, celle du deuxième module devient alors 100 et celle du troisième 1000. Nous pouvons aussi choisir 0,1   1   10, etc.. En principe, toutes les valeurs doivent pouvoir être enregistrées sur le graphique. Avec 100 pour origine, on peut par exemple représenter toutes les valeurs entre 100 et 1000.000 avec 4 modules, ce qui devrait en principe nous laisser une « marge de manœuvre » suffisante quel que soit le problème à analyser. Un inconvénient cependant : il n'est pas possible, en logarithmes, de prendre en compte les valeurs négatives. Une témérarure de – 5 degrés par exemple ne peut pas être représentée sur une échelle logarithmique. Cela ne présente pas un grand inconvénient en statistiqte des affaires. En pratique, si les ventes voisines d'une tonne tombent à zéro, on ne commet pas une erreur grave en enregistrant au lieu du zéro attendu un kilogramme ou même un gramme.

4. Une courbe régulière sur un graphique millimétré devient une droite sur un graphique logarithmique.Ainsi,en etuiant les graphiques de gestion, les isoquantes deviennent des paraleles.

Apres avoir vu les bases de l'utilisation de l'echelle logarithmique nous allons nou pencher sur quelques éléments de nomographie, avant tout en definir des termes.

Nomographie = Technique du tracé des nomogrammes; utilisation des nomogrammes

Nomogramme = Système de courbes permettant de trouver certains résultats par simple lecture, sans calcul.

Abaque = Graphique donnant par simple lecture la solution d'un problème numérique.

*« D'après « Dictionnaire encyclopédique LAROUSSE, édition 2000 ».*

Nomographie = Technique de construction des abaques.

Dans les pages suivantes nous allons faire une petite recomandation de la facon comme on trace et on interprete l'abaque de l'adition. Il se présente comme un quadrillé à double échelles, verticale et horizontale, sur lequel ont été menées des obliques orientées nord-ouest sud-est.

Les obliques, réunissant les mêmes degrés des échelles verticale et horizontale, sont parallèles (Voir Figure #4).

Les obliques représentent la somme des valeurs indiquées sur l'échelle verticale et des valeurs correspondantes de l'échelle horizontale et réciproquement. Par exemple, si nous suivons les droites figurant en caractères gras, nous voyons que le point de rencontre de 7 (échelle verticale) et de 8 (échelle horizontale), se situe sur l'oblique 15.

Figure #4
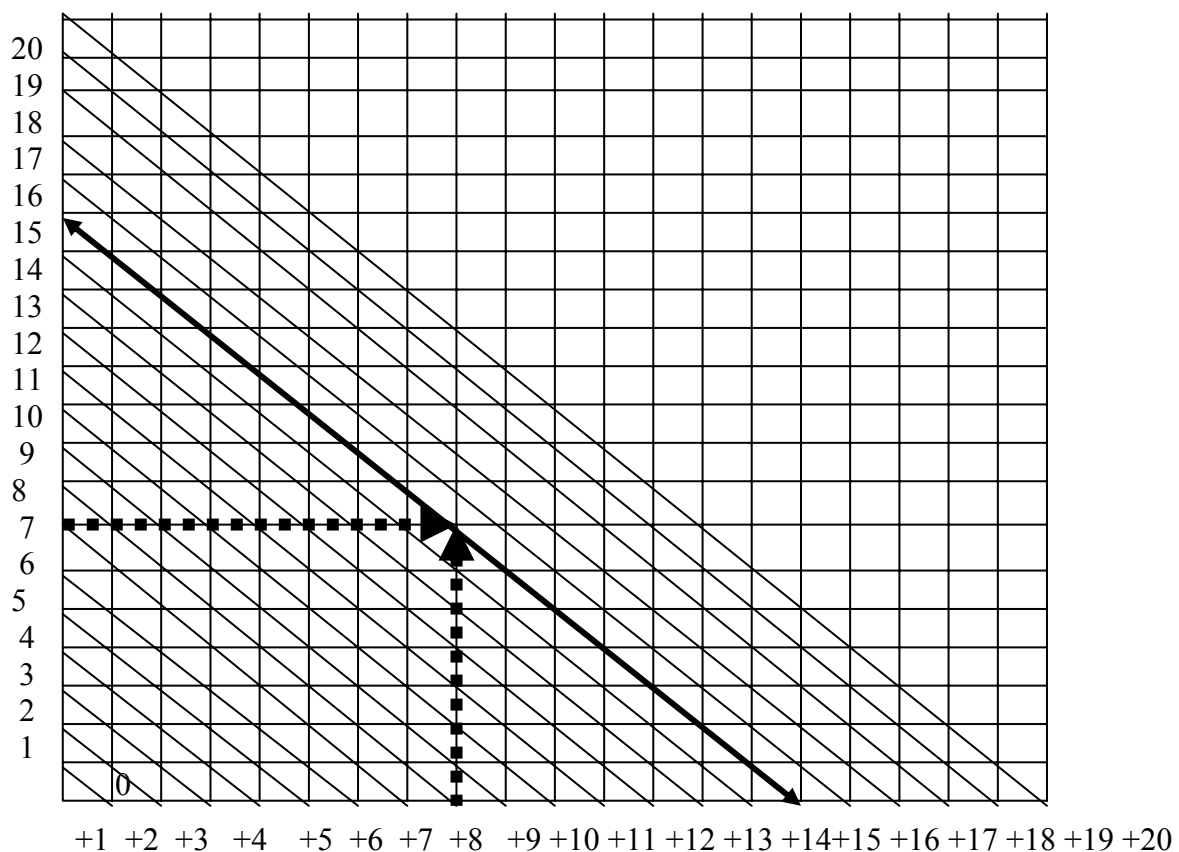
Exemple d'application au domaine commercial - analyse des ventes par produit :

Figure#5
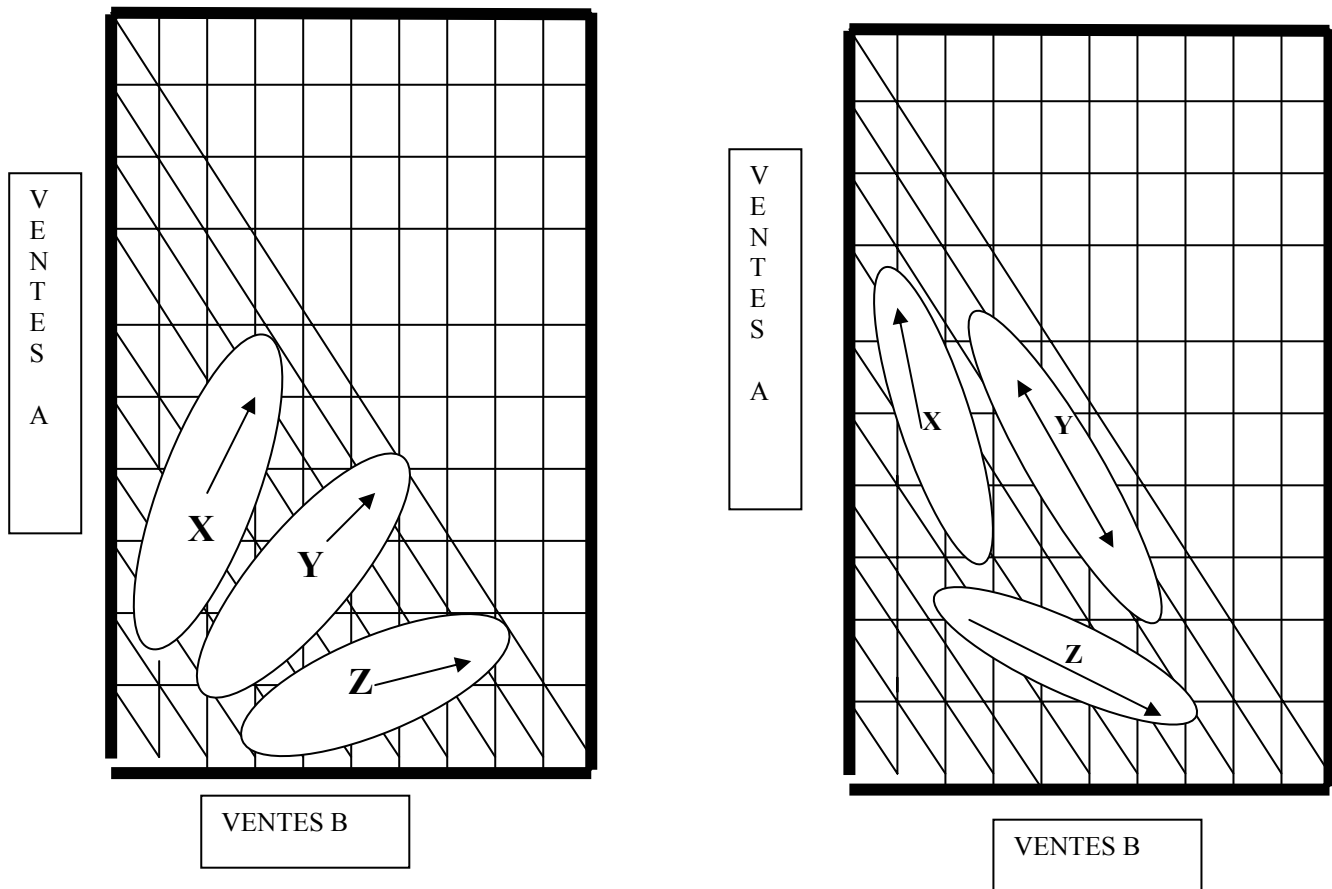
Nos représentants vendent deux types de produits A et B. Sur l'échelle verticale nous considérons les ventes de A, sur l'échelle horizontale les ventes de B. Les obliques nous donnent ventes de A + ventes de B = Ventes totales. Nous obtenons un point par secteur, l'ensemble constituant un nuage de points, dont la direction va permettre d'analyser nos ventes. Si nous obtenons par exemple le nuage Z, nous voyons que lorsqu'on réalise de fortes ventes totales, les ventes de B sont dans l'ensemble supérieures aux ventes de A. Pour le cas où c'est un nuage de type X que nous obtenons, nous constatons le résultat inverse : On obtient les plus fortes ventes en vendant plus de A que de B. Dans le cas du nuage Y, ces ventes sont égales : On vend toujours à peu près autant de A que de B. En conclusion, si nous voulons augmenter notre chiffre d'affaires, nous devrons pousser les ventes de A dans le cas X , les ventes de B dans le cas Z, et dans le cas Y, il ne sera pas nécessaire de promouvoir les ventes de l'un ou de l'autre. Ces méthodes graphiques permettent également de prêter une attention particulière aux points « aberrants », ceux qui se situent en dehors du nuage, et d'en analyser les raisons. Reportons-nous maintenant au graphique IV-1-2. Si nos secteurs de ventes appartiennent au nuage Z, on réalise de fortes ventes totales en vendant beaucoup de produits B et peu de produits A et de faibles ventes totales en vendant beaucoup de A et peu de B. Le nuage X nous montre qu'on réalise de fortes ventes totales en

vendant beaucoup de A et peu de B, et que c'est en vendant beaucoup de B et peu de A qu'on réalise le moins de ventes. Si nos points appartiennent au nuage Y, cela veut dire qu'on obtient à peu près les mêmes ventes totales en poussant indifféremment le produit A ou B.

Pour les abaques de soustraction les choses se passent de la même manière, mais en changeant des diagonales et en ajoutant des valeurs négatives sùr les axes.

L'échelle verticale est celle des « + », l'échelle horizontale étant réservée aux « moins ».

Comme pour l'abaque de l'addition, nous traçons des obliques, mais cette fois orientées direction sud-ouest nord-est.; chaque oblique part d'un degré + ou – d'une échelle, pour aboutir au même degré de l'échelle opposée. Les obliques représentent les valeurs indiquées sur l'échelle verticale – les valeurs correspondantes de l'échelle horizontale. Par exemple, si nous suivons les droites figurant en caractères gras, nous voyons que le point de rencontre de +16 (échelle verticale) et de -13 (échelle horizontale), se situe sur l'oblique +3. L'abaque est aussi valable pour les valeurs algébriques : Le point de rencontre corres- pondant à +6 et à – 20, est bien situé sur l'oblique – 14.

Exemple d'application au domaine financier - analyse des problèmes de facturation:

Beaucoup d'entreprises ont des problèmes de cash-flow réel. Une fois la marchandise expédiée chez le client, on attend plusieurs jours avant d'envoyer la facture, et il est rare qu'à son tour, le client paye dès réception de la facture. Entre la date d'envoi de la marchandise et la date de réception du paiement de la facture, que nous appellerons "*délai total marchandise - paiement*", il peut s'écouler un délai excessivement long, qui souvent pose de graves problèmes de liquidité pour l'entreprise. On va donc considérer dans cette analyse:

Le délai marchandise - facture, dont la longueur est imputable à la lenteur ou à l'engorgement du service facturation de l'entreprise; *Le délai facture – paiement,* dont la longueur est imputable à la bonne ou la mauvaise volonté du client; La somme de ces deux délais donne le temps écoulé entre le départ de la marchandise et le paiement. On peut présenter cette somme sous la forme de l 'égalité: **Délai total = délai marchandise-facture + délai facture-paiement**

Le graphique est construit de la manière suivante (Figure #6) :

**Figure #6**

Graphique IV – 2 – 1
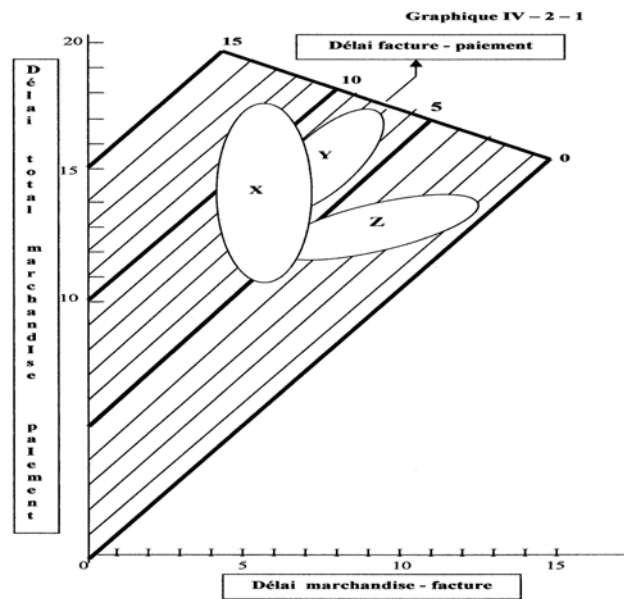
On a porté sur l'échelle verticale le délai total marchandise-paiement (en jours) ; Sur l'échelle horizontale le délai marchandise-facture (délai imputable à l'entreprise) ; Autrement dit : obliques = délai total marchandise-paiement − délai marchandise- facture = délai marchandise-facture (délai imputable au client). Le chef du service facturations prélève au hasard un certain nombre de factures, et relève les différentes dates nécessaires à l'analyse. Chaque facture est donc représentée par un point sur le graphique et l'ensemble des factures constituera donc un ″nuage de points″ en forme d'ellipse, dont l'orientation par rapport aux obliques va nous permettre de cerner notre problème :Supposons que le nuage aille dans la direction de l'ellipse Y (à peu près parallèle aux obliques), cela signifie que le retard dans l'envoi des factures entraîne fatalement un retard de paiement, mais que ce retard est uniquement dû au service facturation de l'entreprise. Le temps mis pour l'envoi des factures n'a aucune incidence sur le comportement de nos clients, qui ne paieront de ce fait ni plus vite ni plus lentement. Ils ont l'habitude de régler les factures à terme fixe. C'est loin d'être le cas si le nuage si les points sont inscrits dans un nuage semblable à X. Ici, plus l'entreprise prend du retard pour l'envoi de la facture, plus le client a tendance à prendre également du retard pour payer! L'augmentation du délai marchandise-facture entraîne une augmentation plus importante du délai facture-paiement, si bien que le délai total marchandise-paiement subit, comme disent les économistes un ″effet multiplicateur″ qui risque de mettre en danger la liquidité de notre entreprise. Il est temps de procéder dans ce cas à une réorganisation de notre sevice de facturation.

En revanche, si notre nuage a tendance à suivre la direction de Z, tout retard dans l'envoi des factures, entraîne raccourcissement du délai de règlement des clients : Ils payent d'autant plus

rapidement que la facture a mis du temps à leur parvenir. Notre antreprise a dans ce cas des clients exceptionnels :Faisons tout notre possible pour les conserver !

Pour obtenir un abaque de la multiplication on a besoin d'un quadrillé logarithmique, on trace des obliques parallèles, orientées Nord-Ouest Sud-Est, et joignant les graduations correspondantes de l'échelle des Y et de l'échelle des X. Les origines des deux échelles peuvent être différentes, tout dépend de la taille des données. Dans l'exemple ci-dessous, l'échelle des X a pour origine 1, alors que l'origine de l'échelle des Y est 10. Si je désire multiplier 20 par 4, je repère la graduation 20 sur l'échelle des Y, 4 sur l'échelle des X, et l'oblique correspondant à leur point de jonction me donne le résultat (sens de la flèche). Le point de jonction correspondant à 30 x me donne l'oblique 150, etc..

Figure#7
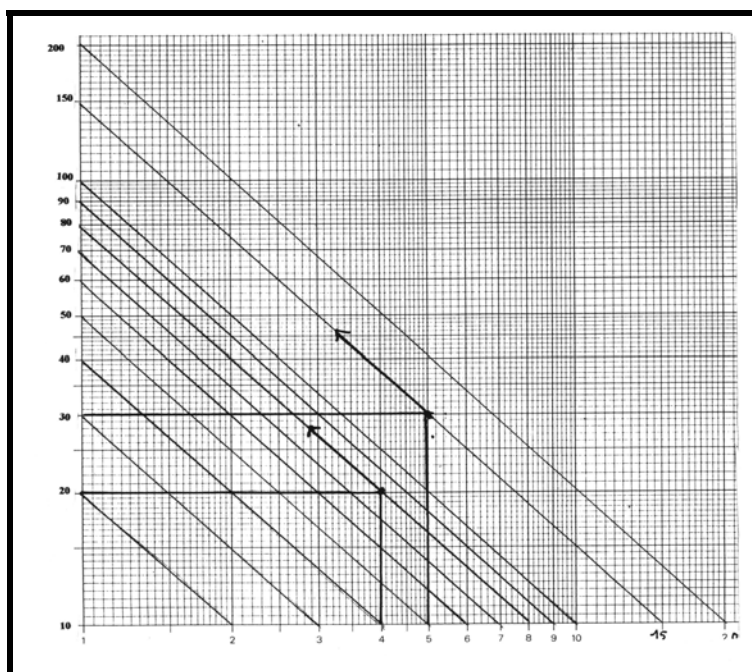
Exemple d'application au domaine commercial - analyse des commandes:

Un thème de réflexion qui est souvent soumis aux responsables commerciaux, est celui de savoir ce qui entraîne les plus fortes ventes: Des commandes peu nombreuses mais importantes? Ou beaucoup de commandes faibles? On observe souvent que les inspecteurs de ventes les plus âgés, du fait de leur expérience et de la connaissance de clients habituels qu'ils connaissent depuis longtemps, auront tendance à passer plus de temps auprès d'eux et d'essayer d'obtenir des commandes importantes : Ce sont des inspecteurs de ventes "intensifs". A l'opposé, on trouve des inspecteurs qui, n'ayant pas encore leurs clients habituels, chercheront plutot à prospecter de nouveaux clients et donc de passer moins de temps avec chacun d'eux. Ils obtiendront peu de commandes importantes, mais amèneront à l'entreprise plus de clients nouveaux. Ce sont des

inspecteurs de ventes "extensifs". Que doit faire le Directeur Commercial ? Opter pour une politique intensive ou extensive des ventes ? Les obliques nous donnent automatiquement :

Nombre de commandes x montant moyen par commande = Ventes

Supposons que le nuage de points suive à peu près la direction des obliques (nuage A – Figure#8) cela signifie que le fait d'avoir beaucoup de petites commandes ou peu de grosses commandes, n'a pas d'incidence sur l'importance de nos ventes. Si le nuage prend une orientation Nord-Ouest, (nuage B) ce sont les commande importantes qui entraînent les meilleures ventes. Le fait d'avoir de nombreuses commandes peu importantes fait baisser nos ventes. Si le nuage prend la direction Nord-Est, (nuage C) c'est le contraire qui se produit : nous avons intérêt à augmenter le nombre de commandes. Dans le cas B, nous devons prôner une politique intensive des commandes. Dans le cas C nous devrons au contraire mettre en place une politique plus extensive.

<u>Figure #8</u>
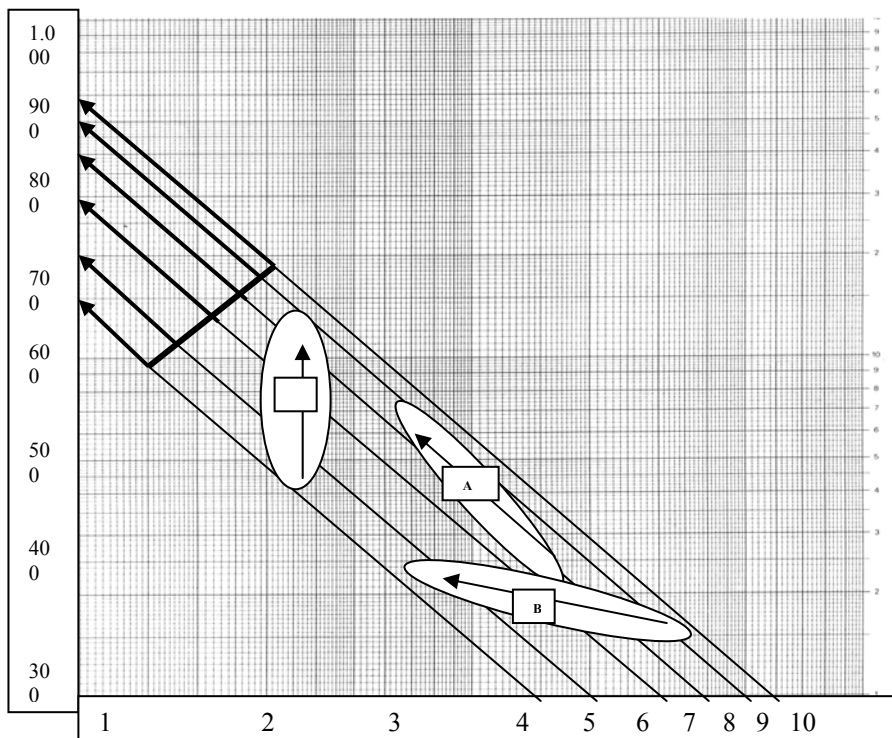
Pour obtenir un abaque de la division il suffit de prendre le graphique ci-dessus et considerer les obliques à orientqtion Sud-Ouest Nord-Est. Il est notable de remarquer que la bisectrice de nos axes nous aide de trouver facilement la racine carrée. On trouve plusieurs applications qui démontre la façon simple de utiliser l'abaque dans les management. Toute formule qui designe une fonction de type $f(x, y) = K \times X \times Y$ peut être réprésenté sr le graphique ; ainsi que les analyses peuvent présenter des conclusionnes toute de suite. Des applications bien intérèsqntes se trouvent dans les domaines des differents indices de productivité, rentabilité, tout ce qui concernne la pérformance dans les affaires :

Conclusion:

Dans notre ouvrage nous avons essayé de démontrer qu'ils existent des nombreux décisions dans le management qui s'appuient sur et valorisent des outils statistiques assez simples. Les abaques de calculs sous leur forme graphique s'adaptent au plusieurs issues de la vie économique de l'entreprise: production, commercial-marketing, financier-comptabilité, etc. Il est nécessaire de concevoir un tableau de bord (une variante logiciel serait bien plus à l'aise) pour rejoindre les abaques, pour les faire „parlantes".

HOW A SME CAN SPIN-OFF
FROM A UNIVERSITY WITH SUPPORT OF EU-PROJECTS

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1. Introduction

BOC Information Technologies Consulting Ltd. was founded in 1995 as a consultancy and software company, specialising in business process management for the financial services sector. Based in Vienna (Austria), BOC additionally operates from offices in Berlin (Germany), Madrid (Spain), Dublin (Ireland), Athens (Greece) and Warsaw (Poland) with a total permanent staff of over 100 employees (April, 2004).

Originated as a spin-off company of the BPMS (Business Process Management Systems) group of the Institute of Applied Computer Science and Information Systems, University of Vienna, BOC establishes close contact to the university. This enables the combination of research results with practical requirements of clients.

The BPMS-paradigm developed by the University of Vienna and BOC Information Technologies Consulting Ltd. provides a framework for business process management. It is based on the conviction that only continuous (re-)organisation leads to success. But living business process management can only be successful if the organisation as a whole accepts business processes as the base of decision. Thus, business process management is a holistic exercise from the mental model of the decision makers to the execution and evaluation of activities. Companies can live this process mentality only if they have skilled and motivated people. Therefore process education and training is vital. But for providing employees the opportune knowledge in the right place at the right time, enterprises must efficiently manage and leverage their knowledge.

Starting from this holistic approach the BOC Information Technologies Consulting Ltd. built up it's research strategy – basically through EU-funded projects. The ESPRIT project Refine provided the chance for demonstrating the usefulness of the BPMS-paradigm as well as the evaluation and further development of the ADONIS®-Toolkit at European level. The follow-up project ADVISOR® aimed, to provide improved access for employees to company and performance related information, to offer the functionality of rapid, semi-automatic production of training materials and to support knowledge acquisition for organisational learning. For managing organisational knowledge the third EU-funded project aimed to build up a toolkit for knowledge management. PROMOTE® guides the accumulation, retrieval and distribution of product-process related knowledge and employees' know-how, and serves as an on-line support tool for knowledge managers as well as for the employees who generate and use knowledge.
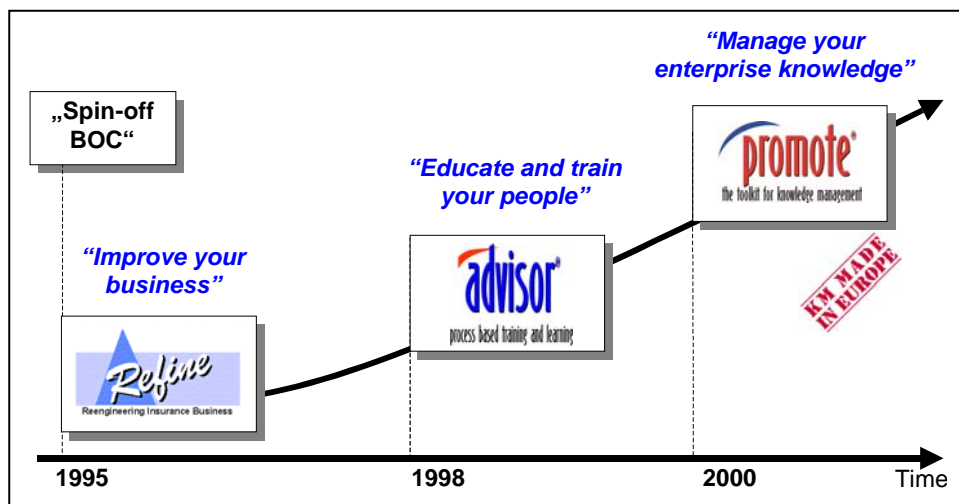

Fig. 1: BOC Ltd. "EU-Project Trilogy"

2. The business process management toolkit ADONIS®

The business process management toolkit ADONIS® was specifically developed to meet the needs of service providers. It provides functionality primarily in the following areas:

- Business Process optimisation/Business Process Reengineering

- Evaluation of business processes (benchmarking, monitoring, comparison of as-is to should-be processes)

- Quality management / ISO 9000 certification and support

- Controlling (process costing)

- Personnel management (capacity planning of personnel and resources)

- Organisational management (organisational documentation, job descriptions, etc.)

- Information management (conceptual development of IT systems, interfaces to workflow and CASE systems, introduction of standard software)

- Migration management (Process-oriented acquisition of existing information systems, development of model-based migration strategies)

- Production of electronic organisational handbooks and their availability on an IntraNet using powerful multimedia features

ADONIS® does not tie the company to a particular method and guarantees the management of business processes at all levels. The core activities of the Re-Engineering Process provide a framework embedding specific methods used within different companies or application scenarios. For this reason, the ADONIS® components have been developed according to the core activities of the Re-Engineering Process:

- Information Acquisition
- Modelling
- Analysis
- Simulation
- Evaluation
- Import/Export
- Transformation

The additional available modules are:
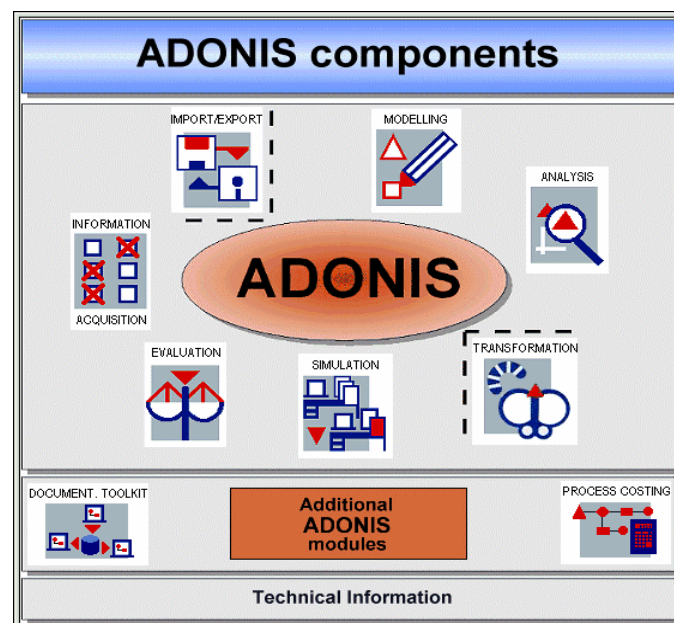
- Documentation toolkit
- Process costing component

Fig. 2: Components of the ADONIS® Toolkit

3. ADVISOR®: A Process-Integrated Learning and Training Framework

In ADVISOR®, software and methods have been developed to enable employees of three European insurance companies to learn more about their business processes and to relate the idea of process orientation to their own work. ADVISOR® integrates training and business process modelling to avoid a fragmentation of conceptualisation and understanding of corporate objectives and tasks, and to treat process optimisation and learning as one coherent developing system.

Usually, information about the organisation and its processes is captured to allow for reengineering, rationalisation, quality-management and application-development. However, existing process documentations are hardly usable for employees in their daily job, because they are not adapted to the needs and language of employees in operational business. ADVISOR® tries to overcome this by providing employees with an intranet information resource that can be used for learning and training as well as for performance support. The resource is easily adaptable to changes and therefore provides up-to-date information.

The approach focuses on transforming individual learning into corporate learning by:

- Relating learning activities to business processes
- Making process orientation a salient corporate value
- Delivering mission critical knowledge to every single employee
- Providing information in a meaningful context
- Enabling people to identify the information that is relevant for their own personal contribution to the process chains
- Allowing the identification of qualification requirements based on process models
- Avoiding unnecessary disturbance of operational business by providing on the job learning and training.

4. PROMOTE®: Process oriented knowledge management

PROMOTE® aims to become an enabler, a support tool for knowledge-intensive work in highly innovative environments (like insurance and bank services).

The overall goal of PROMOTE® was to adapt the existing "Business Process Management Systems" methodology for realising "Process oriented knowledge management". PROMOTE® guides accumulation, retrieval and distribution of product-process related knowledge and employees' know-how, and serves as an on-line support tool for knowledge managers as well as for the employees who generate and use knowledge.

Knowledge intensive work has different characteristics from other ordinary activities. Employees while using PROMOTE® and giving their feedback will gain the experience (e.g.

corrections, update) of being the company's main capital. Employee empowerment is a natural consequence, increases in specific qualifications but also in general ones that as competences in self-guided information search and learning will raise employability.

PROMOTE® will improve current working condition. When using the PROMOTE tool employees do not any longer loose time or feel frustrated when they are looking for another persons' expertise in the company.

PROMOTE® is based on multidisciplinary research: computer science, business administration, psychology and organisational research are combined. Today, multidisciplinary work is seen as a necessary approach to handle modern complex socio-economic and technological problems.

MATHEMATICS VERSUS BIOLOGY IN NEURAL NETWORKS.

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Abstract: *Neural computation is inspired by knowledge from neuroscience, though it does not try to be biologically realistic in details. The artificial model of a biological neural network and the learning capability is improved using mathematical strategies. Neural networks can learn, absorbing experience, modifying their internal structure in order to accomplish a given task. In the classical forms of supervised learning, the training set is chosen according to some known or random given distribution. The trainer is a passive agent in the sense that he is not able to interact with the training set in order to improve the performances of the neural networks learning. We will investigate some mathematical possibilities that allow the trainer to become active and we will analyse the performances of such supervised learning.*

INTRODUCTION

Neural computing represents an alternative computational paradigm to the algorithmic one (based on a programmed instruction sequence). Neural computation is inspired by knowledge from neuroscience, though it does not try to be biologically realistic in details [9].
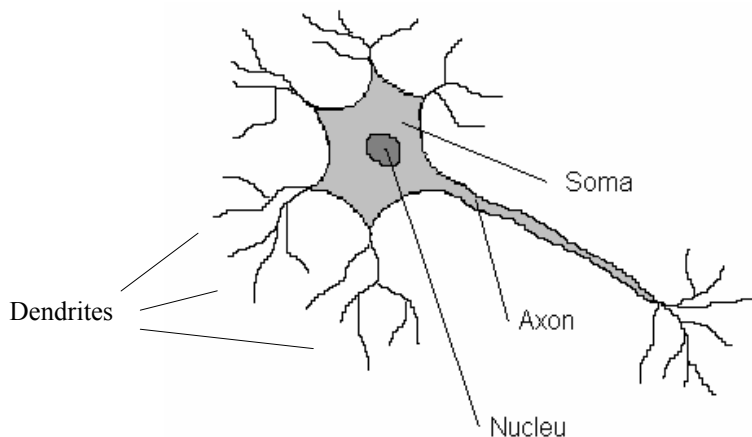

Fig. 1. Schematic drawing of a biological neuron

The brain is composed of about 10^{11} neurons of many different types. In Fig. 1 we have represented a schematic drawing of a single neuron. Tree-like networks of nerve fibre called dendrites are connected to cell body or **soma**, where the cell nucleus is located. Extending from the cell body is a single long fibre called the **axon**, which eventually branches or **arborizes** into strands and sub-strands. At the ends of these are transmitting ends of the **synaptic junctions**, or the **synapses**, to other neurons.
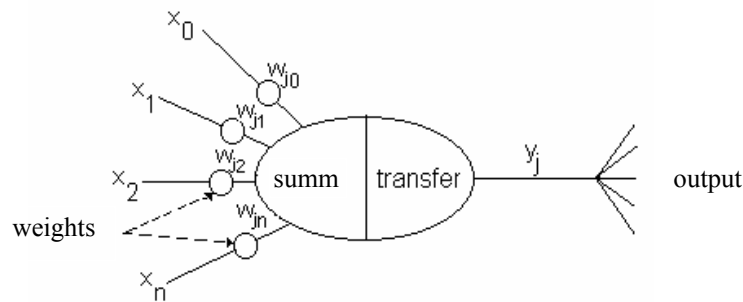

Fig. 2. Schematic diagram of an artificial neuron

An artificial neuron has several inputs, say n. Each input arrives from another neuron. Each input has a connection strength associated with it. For a given neuron, call it neuron j, the connection strength on the input coming from neuron i is written as w_{ji} (See Fig.2.). The artificial neuron performs the following operations:

➲ a summation of weighted inputs:
$$\sum_i w_{ji} \cdot x_i$$

➲ a non-linear thresholding of this sum:
$$f\left(\sum_i w_{ji} \cdot x_i\right)$$

➲ the output of this neuron, j, is:
$$y_j = f\left(\sum_i w_{ji} \cdot x_j\right)$$

These neurons can be connected together in many ways. The particular manner of interconnection is called **architecture** of the network.

We will deal with neural networks organised in layers, where the information is transmitted from the first layer until the last layer. These types of feedforward multylayer neural networks are called multilayer perceptrons [9]. Some important fact about artificial neural networks:

➲ the first layer of neurons is called input layer, it is a simple buffer to store the input data.

➲ the input signal is transmitted to the connected (hidden) neurons.

➲ the last layer is the output of the system, and is usually called output layer.

Multilayer perceptrons learn in a supervised manner [21]. Learning represents the process in which input patterns are presented repeatedly and the weights are adjusted according to the learning

algorithm, which in this supervised case take the difference between the desired output and the current output into consideration.

MATHEMATICAL ASPECTS OF SUPERVISED LEARNING

The supervised learning of the neural networks uses a training set has the following form:

$$T = \left\{ \left(\mathbf{x}_i, \mathbf{z}_i \right) \mid i = 1, 2, \ldots, N \right\} \tag{1}$$

where $\mathbf{x}_i \in \mathbf{R}^n$ is the n-dimensional input vector, and $\mathbf{z}_i \in \mathbf{R}^m$ is the m-dimensional target vector that is provided by a trainer. $N \in \mathbf{N}$ is a constant that represents the number of training samples. Usually the training set T is obtained from a probabilistic known distribution. In the classical supervised learning strategy [9], [16], [20] the trainer is a static agent. Using the probabilistic distribution he selects a certain input vector $\mathbf{x}_i$, and provides the appropriate target vector $\mathbf{z}_i$. The learning algorithm will compute the difference between the output generated by the neural network $\mathbf{y}_i$ and the desired target vector $\mathbf{z}_i$, which will represent the error signal:

$$e_i = y_i - z_i, i = 1, 2, \ldots, N \tag{2}$$

The signal error is used to adapt the synaptic weights w_{ji} using a gradient descendent strategy [9]:

$$w_{ji} = w_{ji} + \eta \frac{\partial E}{\partial w_{ji}} \tag{3}$$

where $\eta \in (0,1)$ is the learning rate, controlling the descent slope on the error surface which is corresponding to the error function E:

$$\mathrm{E} = \frac{1}{2} \sum_{i=1}^{N} \left(y_i - z_i \right)^2 \tag{4}$$

Consider a physical phenomena described by a vector $\mathbf{x} \in \mathbf{R}^n$ which corresponds to a set of independent random variables, and a real[1] number $z \in \mathbf{R}$ that represents a dependent variable. Let's consider that we have N distinctive measurements (observations) of variable $\mathbf{x}$:

$$\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \ldots, \mathbf{x}_N \tag{5}$$

and a corresponding set of scalars z:

$$z_1, z_2, z_3, \ldots, z_N \tag{6}$$

Usually we do not have enough information about the exact relationship that exists among the variable $\mathbf{x}$ and z. For this reason we will consider a relation represented by the following equation:

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[1] A neural network with m output neurons can be considered as m distinct neural networks with only one output neuron. This is why it is allowed to consider, without loosing the generality, a neural network with a single output

$$z = f(\mathbf{x}) + \varepsilon \qquad (7)$$

where f is a function which depends on the variable $\mathbf{x}$, and ε is the error represented by a random variable. The error ε represents the mistake made in order to estimate the existing functional relation between variables $\mathbf{x}$ and z. Equation (7) is a known statistical model, named regressive model.

Using statistical relations [22], we are now able to express the function f of the regressive model as:

$$f(\mathbf{x}) = E[z|\mathbf{x}] \qquad (8)$$

where $E[z|\mathbf{x}]$ represents the conditional statistical average, namely, we will have on average the target value z if he has a particular realisation of variable $\mathbf{x}$. In particular, if the functional relation between variables $\mathbf{x}$ and z is known precisely, then we can consider in the regressive model the ideal case $\varepsilon = 0$.

The main feature of this regressive model is the prediction of variable z using the input variable $\mathbf{x}$. The above interpretation is just a mathematical interpretation, and let us analyse what kind of meaning it has from the neural computing point of view.

A neural network is a physical mechanism to implement what we have stated in the regressive model: the prediction of z on the bases of $\mathbf{x}$. This main goal is achieved by encoding the information content in the training set (1) in the synaptic weights w_{ji}. It is quite clear that from the neural computing point of view $\mathbf{x}$ represents the input vector presented at the input layer, and z represents the target vector that we wish to obtain at the output layer of the neural network.

We will note with $\mathbf{w}$ the synaptic weights vector of the neural network that is supposed to approximate the regressive model (7). By applying the input vector to the input layer of the neural network, and by propagating it through the output layer we can write the following equation [17]:

$$y = F(\mathbf{x}, \mathbf{w}) \qquad (9)$$

Because the training set $T = \{(\mathbf{x}_i, z_i) \mid i = 1, 2, \ldots, N\}$ contains also the target vectors $\mathbf{z}$ provided by the trainer it is clear the equivalence with the supervised learning paradigm. For this reason the modification of the synaptic weights is done using an iterative process, as a response to the error signal (2). In Fig. 3 we have represented the interpretation of the regressive model from the neural computing point of view [23].
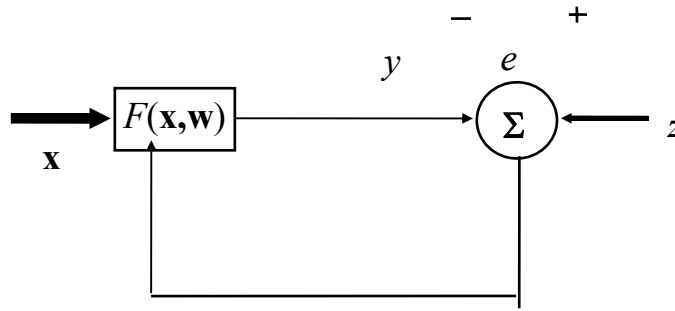
---

neuron instead of a neural network with m output neurons. In conclusion, we are allowed to consider, when it is necessary $y, z \in \mathbf{R}$ instead of $\mathbf{y}, \mathbf{z} \in \mathbf{R}^m$.

Fig. 3. Neural computing representation of the statistical regressive model

The supervised learning algorithm will optimise the following error function, in respect with the synaptic weights **w** of the neural network:

$$\mathbf{E}(\mathbf{w}) = \frac{1}{2} E\!\left[e^2 \right] = \frac{1}{2} E\!\left[(z - y)^2 \right] = \frac{1}{2} E\!\left[(z - F(\mathbf{x}, \mathbf{w}))^2 \right] \qquad (10)$$

This mathematical background related to supervised learning would not be complete if we do not outline the fact that a neural network is a universal approximator of any continuos functions [18]. The architecture of such a neural network is equivalent to a multylayer perceptron having three layers (an input layer, an output layer and a hidden layer). The activation function of the neurons in the hidden layer must be any non-polynomial function and the activation function of the neuron (neurons) in the output layer can be a linear function or an average function of the output values of the hidden neurons. From this point of view a neural network is an approximation scheme that permits us to approximate at any accuracy any continuos function, provided we have an enough number of hidden neurons [19]. The approximation is obtained through the supervised learning process, which is based on an iterative modification of the synaptic weights of the neural network. Presenting repetitively the training set we will be capable to attain a good generalisation power with the neural network.

NEURAL NETWORKS ARE UNIVERSAL APPROXIMATORS

The question that arises is whether multilayer feedforward perceptrons are in fact inherently limited to approximate only some special class of functions or they are universal approximators. This approach prompts a natural problem that has to be solved if we want to construct a multilayer feedforward neural network [1]:

- Given a function *f,* is it possible to implement it by a multilayer feedforward neural network? Or at least, if it is impossible to implement it precisely, is it possible to approximate this given *f* function with a given ε precision [13]?

In the literature we can find many important results concerning the capability of the neural networks to be universal approximators [2], [3], [4], [5], [6], [7], [8].

The approximation problem can be formulated formally as [12], [14]:

if $f: \mathbf{X} \subseteq \mathbf{R}^n \to \mathbf{R}^m$ *is a continuos function and* $F_\mathbf{w}: \mathbf{R}^n \to \mathbf{R}^m$ *is an approximator function that depends continuously on* $\mathbf{w} \in P$, *determine the parameters* $\mathbf{w}^*$, *such that:*

$$d\left[F(\mathbf{x}, \mathbf{w}^*), f(\mathbf{x})\right] \le d\left[F(\mathbf{x}, \mathbf{w}), f(x)\right], (\forall) \, \mathbf{w} \in P$$

Typical results deal with the possibility, given a network, of approximating any continuos function arbitrarily well. In mathematical terms this means that the set of functions that can be computed by the network is **dense**. We can build neural networks such the corresponding set of approximating function is dense in $C[\mathbf{R}]$. We will consider the following general neural network:
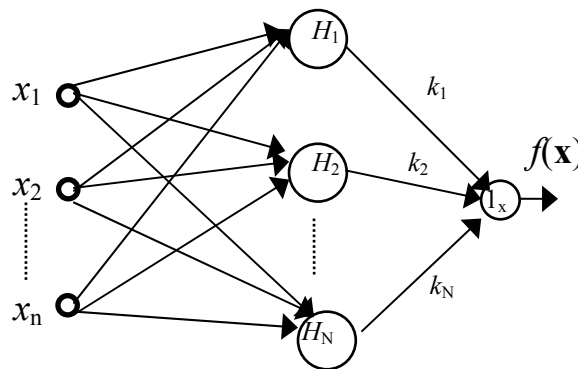

Fig 4. A general feedforward neural network that is a universal approximator

The architecture of such a neural network is composed of:
- an input layer with n input neurons;

- a hidden layer with N hidden neurons and the activation functions $H_i(\mathbf{x}) = e^{-\frac{(\mathbf{x} - \mathbf{t}_i)^2}{\sigma_i^2}}$, where $\mathbf{t}_i \in \mathbf{R}^n$ are the centres of the clusters of the input data and σ_i represents the radius of the clusters. This type of functions are called Gaussian and the corresponding neural networks are called RBF (Radial Basis Functions) neural networks [20].

- the weights $\mathbf{w}_i$ from the input layer to the output layer are included in the special form of the activation function H_i of the i-th hidden neuron.

- an output layer having only one output neuron with the identity function $I_x(\mathbf{x}) = \mathbf{x}$ as activation function;

- the hidden neurons are connected to the output neuron by the weights $k_i \in \mathbf{R}$;

The class of approximating functions corresponding to it is:

$$F = \{F \in C[U] \Big| F(\mathbf{x}, \mathbf{w}) = \sum_{i=1}^{N} k_i \cdot H_i\big(\mathbf{x}; \mathbf{w}_i\big), U \subseteq \mathbf{R}^n, H \in C[U], N \in \mathbf{N}\}$$

The Stone-Weierstrass [10], [11] theorem can be used to show that certain network architectures possess the universal approximation capability.

Theorem (*Stone-Weierstrass*): Let domain U be a compact space of n dimensions, and let F be a set of continuos real-valued functions on U, satisfying the following criteria:

(C1) Identity Function: The constant function $f(x) = 1$ is in F;

(C2) Separability: For any two points $x_1 \neq x_2$ in U, there is a function $f \in F$ such that

$$f(x_1) \neq f(x_2);$$

(C3) Algebraic closure: if f, $g \in F$, then fg, $af + bg \in F$ $(\forall)$ $a, b \in \mathbf{R}$;

Then F is dense in $C[U]$, the set of continuos functions on U. In other words, $(\forall)$ $\varepsilon > 0$ and $(\forall)$ $g \in C[U]$ $(\exists)$ $f \in F$ such that $|g(x) - f(x)| < \varepsilon$.

DIFFICULT LEARNING AND ACTIVE SUPERVISED LEARNING

In our presentation of the supervised learning we have outlined the equivalence between the statistical regressive model and the supervised learning of a feedforward neural network. But the trainer has a passive role in the learning process being a simple recipient of passive information about the target function (function to be approximated).

We want to determine, if we cam consider a more active role for the trainer in the learning process, so, instead of giving only the target vector z for a specific input vector x, to try to indicate which input vector should be selected from the training set, in order to improve the learning capabilities of the neural network, which is equivalent to approximate better with the neural network F the target function f. We can consider that for a specific target function f we have some areas where the function is more "difficult" to be learned (approximated) and so the trainer should choose more examples in order to reduce the approximation error.

In conclusion, we can speak about "difficult" and "easy" regions where the target function is approximated. A "difficult" region will be considered a region with a high approximation error and an "easy" region will be considered a region with a low approximation error (close to zero). This definition is not a very rigorous one because we did not establish the limit that delimits "high approximation error" from "low approximation error". But we will see in the next pages that these definitions are not so important, because from the learning algorithm that we will consider, "high approximation error" will be considered the maximum error obtained on the regions which constitute the definition domain of the target function f.

It is obvious that our active learning is based on a fundamental assumption: the trainer is allowed to choose his own examples in order to accomplish the task of approximating the target function as well it is possible. In this assumption the trainer should be capable to decide which are the "difficult" regions to be learned and the learning samples from those "difficult" regions.

We will compare the passive supervised learning with the active supervised learning, keeping unchanged the other parameters that influence the learning process. In this approach the only difference between passive and active learning consists only in the way the learning examples are chosen. Another goal of our paper will be to develop a general frame for choosing the examples for the approximation of real functions. We will make also some simulations in order to prove the validity of the theoretical results presented here.

We will need to introduce the following terms:

- F the set of functions defined on set D with values in set Y, where $Y \subseteq \mathbf{R}$.

$$F = \left\{ f : D \subseteq \mathbf{R}^n \to Y \subseteq \mathbf{R} \right\} \qquad (10)$$

The target function f that should be approximated by an approximation scheme (a neural network) belongs to the set F of functions.

- The training (learning) set T is composed by pairs of elements:

$$T = \left\{ (\mathbf{x}_i, z_i) \,\middle|\, \mathbf{x}_i \in D, \; z_i = f(\mathbf{x}_i), \; i = 1, 2, \ldots, N \right\} \qquad (11)$$

- H is an approximation scheme. This means that H does not contain only a set of functions defined on the set D with values in the set Y, but also the algorithm what the trainer is using to choose the approximator function $F \in H$, based on the learning set T. In other words we will denote by H a couple $\langle H, A \rangle$, where H is a the set of functions from where we will chose the approximator function F, and A is an algorithm which has as input the learning set T, and generates at the output the approximator function $F \in H$.

- d_C will represent a metric that measures how good is the approximation made by the trainer. More precisely, the metrics d_C measures the error on a subset $C \subseteq D$. This metrics will have the following properties:

  $\forall \, C_1, C_2 \subset D, \; C_1 \subset C_2, \; d_{C_1}(f_1, f_2) \le d_{C_2}(f_1, f_2);$

- $d_D(f_1, f_2)$ is the distance between two functions on the whole definition set D; it represents the basic criterion to measure the quality of approximation.

Example 1: If we consider real functions defined on $\mathbf{R}^n$ with real values, an example of a metric that can be considered is the well-known metric L_C^p, defined as:

$$d_C(f_1, f_2) = (\int_C |f_1 - f_2|^p dx)^{\frac{1}{p}} \tag{12}$$

- C represents a partition of domain D. We will suppose that all the data points from domain D, which will be chosen to approximate the target function f, partition the domain D in a number of disjoint sets $C_i \in C$, $\bigcup_{i=1}^{N} C_i = D$.

Example 2: For functions defined on a real interval, a normal way to consider a partitioning set C of domain D, is to take intervals having the following form: $[x_i, x_{i+1})$ or $(x_i, x_{i+1}]$ or $[x_i, x_{i+1}]$.

The trainer's main objective can be stated as follows: operating with an approximation scheme H, based on the leaning set T, obtain the approximator function $F \in H$ of the target function f. In the literature the most widely used criteria to measure the performance of the learning algorithms is the PAC criteria (**P**robably **A**pproximately **C**orrect) [24]. We can reformulate the general PAC criteria for our purposes, obtaining the following definition [25]:

Definition 1: We will denote an approximation schema as PAC learnable for a function $f \in F$, if for any $\varepsilon > 0$, for any $1 > \delta > 0$ and for any random probabilistic distribution P on domain D, the approximation scheme can choose a training set $T \subseteq D$, and based on this set can compute the approximation function $F \in H$, so that $d_T(F, f) < \varepsilon$ with a probability greater than $1 - \delta$.

Definition 2: The function class F is PAC learnable if there is an approximation schema which is capable to learn PAC any function $f \in F$.

Definition 3: The function class F is independent PAC learnable, if it is PAC learnable for any probabilistic distribution P defined on domain D.

It is important to outline that the metric D used in the above definitions is arbitrary. This metric does not have to be related to the probabilistic distribution P that generates the training data from the training set T. This is something new for the PAC schemes.

ALGORITHMS FOR ACTIVE LEARNING

In the previous paragraphs we have introduced the concepts of passive supervised learning. As we have mentioned, in this case of passive supervised learning, the trainer is supposed to choose his training samples according to some probabilistic distribution defined on domain D. If the passive learning process will be successful, the neural network implemented to learn the training

samples will correspond to an approximator function *F,* so that we have to obtain the relation $d_C(F, f) < \varepsilon$ with a probability greater than $1 - \delta$. In other words, using Definition 1, we can say that the neural network that is the approximator schema learned PAC the target function *f.*

As an alternative, in the active learning the training will have the possibility to choose according to some strategy the training examples from the domain D where the target function *f* is defined. At a certain moment in learning process, the training set will contain some valuable information about the target function f that has to be approximated by the means of the neural network. Particularly, the training set contains information about the "difficult regions" to be learned, where there is a "high" approximation error. Of course, the trainer will choose more examples in this "difficult region", in order to decrease the total approximation error. In conclusion, we have to develop a learning strategy that will be active in the sense that the trainer can decide which will be the next training sample in the learning process.

First, let's establish the mathematical arguments that describe the mechanism of active learning. Considering the domain *D*, the trainer can access all the data from the following general training set:

$$T = \left\{ (\mathbf{x}_i, z_i) \,\middle|\, \mathbf{x}_i \in D, \, z_i = f(\mathbf{x}_i), \, i = 1, 2, ..., N \right\} \tag{13}$$

The approximation schema *H* (the neural network), after the learning process, will generate an approximator function $F \in H$, using a learning algorithm A that corresponds in the best way to the training set..

We will use also the following notations:

- $C = \left\{ C_1, C_2, ..., C_p \right\} C_i \subset D, \, i = 1, ..., p_N{}^2$, a partition of domain *D*;

- $F_T = \left\{ f \in F \,\middle|\, f(\mathbf{x}_i) = z_i, \forall (\mathbf{x}_i, z_i) \in T \right\} \tag{14}$

The functions belonging to the class of functions F_T are the functions that are passing through the points of the training set *T*. Evidently, the target function is a member of the set F_T.

We are now capable to define the following error criteria [25]:

$$e_C(H, T, F) = \sup d_C(F, f), f \in F_T \tag{15}$$

The meaning of this error is very important. $e_C(H, T, F)$ measures the maximum error of the approximation schema (neural network) on the region *C*. This error is dependant on the training set and on the class of functions to which the target function belongs. As we can see it does not depend directly on the target function (function to be approximated), but we cannot forget that this

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2 The number *p* of regions in which the domain *D* is partitioned by *N* points depends on the specific geometry of domain *D*. For example, if *D* is a real interval then $p = N + 1$

dependency is already captured in the training set, so the target function is present in the above equation.

In this moment we have a certain measurement schema of the uncertainty on different regions of the domain D. In other words, we have now the possibility to define what a "difficult region" for leaning is. From now on, we will consider a ***difficult region for learning*** a region C_i that has the biggest error according to equation (15).

In this way, we have a natural approach for active learning:

Choose the next training sample from the difficult region for leaning

Lets suppose we define the procedure that gives us the possibility to choose the next training sample from the most difficult region for learning with P. This procedure can be very simple:

Procedure P: Choose the sample as the gravity centre of region C_i that is the most difficult region for learning.

Of course this procedure can be adapted to the wishes of the trainer and to the particular form of the target function f.

If we are approximating a one dimensional real function, as we have seen before a region is an interval $C_i = [x_i, x_{i+1}]$, then the next training sample will be:

$$x_{new} = \frac{x_i + x_{i+1}}{2} \qquad (17)$$

We have now a possible active strategy for supervised learning. Lets suppose that at one moment the trainer has obtained the new training sample $\mathbf{x}_{new} \in D$. The next thing the trainer will want to know will be the value of the target function in this point. This value will belong to the following data set:

$$F_T(\mathbf{x}) = \left\{ f(\mathbf{x}) \middle| f \in F_T \right\} \qquad (18)$$

If the requested value is $z \in F_T(\mathbf{x})$, in this moment the trainer has a new supervised training pair $(\mathbf{x}_{new}, z)$ which can be added to the existing training set T, obtaining a new training set:

$$T^* = T \cup \left(\mathbf{x}_{new}, , z \right) \qquad (19)$$

The approximation schema H can now reconsider the approximator function F^*, on the basis of the new training set T^*. We have:

$$e_C(H, T^*, F) = \sup(F^*, f), f \in F_{T^*} \qquad (20)$$

In this moment the error $e_C(H, T^*, F)$ represents the highest possible error related to the new training set T^*. When we choose the new training sample $\mathbf{x}_{new} \in D$ we do not know if we have the necessary information about the value of the target function in this point. A possible strategy to avoid this problem is to choose the "worst case", namely the value which is producing the highest error if $\mathbf{x}_{new} \in D$ is the new training sample.

With this approach, the total error on domain D will be:

$$\sup_{z \in F_T(\mathbf{x})} e_D(H, T^*, F) = \sup_{z \in F_T(\mathbf{x})} e_D\left(H, T \cup \{\mathbf{x}_{new}, z\}, F\right) \qquad (21)$$

Our intention is to obtain the training sample that minimises the maximal error. In this respect the new training sample should be chosen according to the following formula:

$$x_{new} = \arg \min_{\mathbf{x} \in D} \sup_{z \in F_T(\mathbf{x})} e_D\left(H, T \cup \{\mathbf{x}, z\}, F\right) \qquad (22)$$

Using this strategy we are now able to define the following algorithm for active supervised learning. This algorithm gives the trainer the necessary tool to choose the optimal training samples that will improve the supervised learning performances of the approximation schema (neural network).

Step 1: $j := 1$. Choose the first training sample $(\mathbf{x}_j, z_j)$ according to procedure P.

Step2: Based on the new training example, partition domain D in the regions $C_1, C_2 ..., C_{p_j}$.

Step 3: Compute the errors e_{Ci}, for every $i = 1, 2, ..., p_j$.

Step 4: Suppose at ***Step j*** domain D is partitioned in the regions $C_1, C_2 ..., C_{p_j}$. According to procedure P. we will choose in the most difficult region for learning the new training point $\mathbf{x}_{new} \in D$. Lets consider the new training sample $(\mathbf{x}_{j+1}, z_j) := (\mathbf{x}_{new}, z)$.

IF $e_D(H, T, F) < \varepsilon$ **THEN**

$\qquad\qquad$ {

$\qquad\qquad\qquad$ N := j;

```
          exit;
    }
    ELSE
    {
          j: = j+1;
          GOTO Step 2;
    }
```

An important calculation is made in our algorithm to obtain the error over the entire domain e_D (*H, T, F*). This error represents a measure of the highest possible error made by the approximation schema (neural network) in order to approximate a target function from *F*, using the training set *T*. If we want an independent approximation schema we have to minimise the error e_D (*H, T, F*) relative to all the possible approximation schemes:

$$\inf_H(H, D, F)$$ (23)

Another possibility should be to make the error e_D (*H, T, F*) independent from the training set. Having an approximation schema *H* and an arbitrary training set *T,* we can calculate the probability:

$$P\{e_D(H, T, F) > \varepsilon\}$$ (24)

In order to make the approximation schema independent from the training set:

$$P\left\{\inf_H(H, T, F) > \varepsilon\right\}$$ (25)

EXPERIMENTS, SIMULATIONS AND CONCLUSIONS

We have implemented a RBF neural network having is architecture as the one presented in Figure 4. The learning process was carried out in two phases [19], [21]:

- Unsupervised learning phase [15] in order to determine the following unknown parameters:

$t_i \in \mathbf{R}^n$ the centres of the clusters of the input data and σ_i the radius of the clusters.

- Supervised learning phase in order to determine the synaptic weights $k_i \in \mathbf{R}$.

The supervised learning phase was done using three different types of training:

1. **Random passive** - the training set was generated randomly from the domain *D*.

2. **Uniform passive** - the training set was generated using a uniform distribution on domain D.

3. **Active** - the training set was determined using the active learning algorithm presented earlier in this paper.

The experiments were made in order to approximate the following target function:

$$f:[0,1] \rightarrow \mathbf{R}, \quad f(x) = \left(x - \frac{1}{3}\right)^3 + \frac{1}{27} \tag{26}$$

The training set generated by one of the three methods was presented repeatedly in epochs of 1000, 5000 and 10.000 times.

Analysing the learning performances we will take in consideration not only the learning error E_l but also the generalisation error E_g that correspond to the error generated by the neural networks in points which does not belong to the training set. This generalisation error is the real measure of comparing the performances of different approximation schemes.

Nr. of epochs	random passive	uniform passive	active
1.000	$E_l = 0.00111933647$ $E_g = 2.00543792e\text{-}5$	$E_l = 0.00538671535$ $E_g = 9.25674175e\text{-}5$	$E_l = 0.005939686434$ $E_g = 0.000305306076$
5.000	$E_l = 0.00042799210$ $E_g = 6.62271543e\text{-}6$	$E_l = 6.77417526e\text{-}6$ $E_g = 1.19316687e\text{-}6$	$E_l = 8.411126178e\text{-}5$ $E_g = 1.167369815e\text{-}5$
10.000	$E_l = 0.00039366463$ $E_g = 5.98244244244$	$E_l = 5.59375032e\text{-}5$ $E_g = 1.01227192e\text{-}5$	$E_l = 5.386507373e\text{-}5$ $E_g = 5.824087429e\text{-}7$

Table 1: Results of the learning process to learn the target function with N = 100 training data samples, 25 centres using random passive, uniform passive and active supervised learning.

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UNIDIMENSIONAL ANALYSIS MODEL (ANOVA)

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Abstract: *With SASM I experimented on a data base taken out from the primary evidence of a medical unit in Drobeta Turnu Severin; that data base contained 1791 persons as potential and effective patients of the sanitary unit.The most of the obtained data supposted the statistical analysis by the aid of the SASM program.*

The scope of information about phenomena in the medical field could be extended by using some more developed methods in the frame of the analitical process pending the statistical inference, especialy the models of variational analysis ANOVA (the uni-dimensional model for variational analysis).

In the factorial analysis of the social – economical phenomenon a serie of variational analysis statistical models are used on a large scale for identification of the dependences between variables and the causes links can be quantified from these phenomenon.

One of the practical ways of using of these models is variance analysis (ANOVA). In the following we present the main elements of methodological order of this method.

In this following picture is presented the general scheme of basement elements of variance analysis for unidimensional serie.
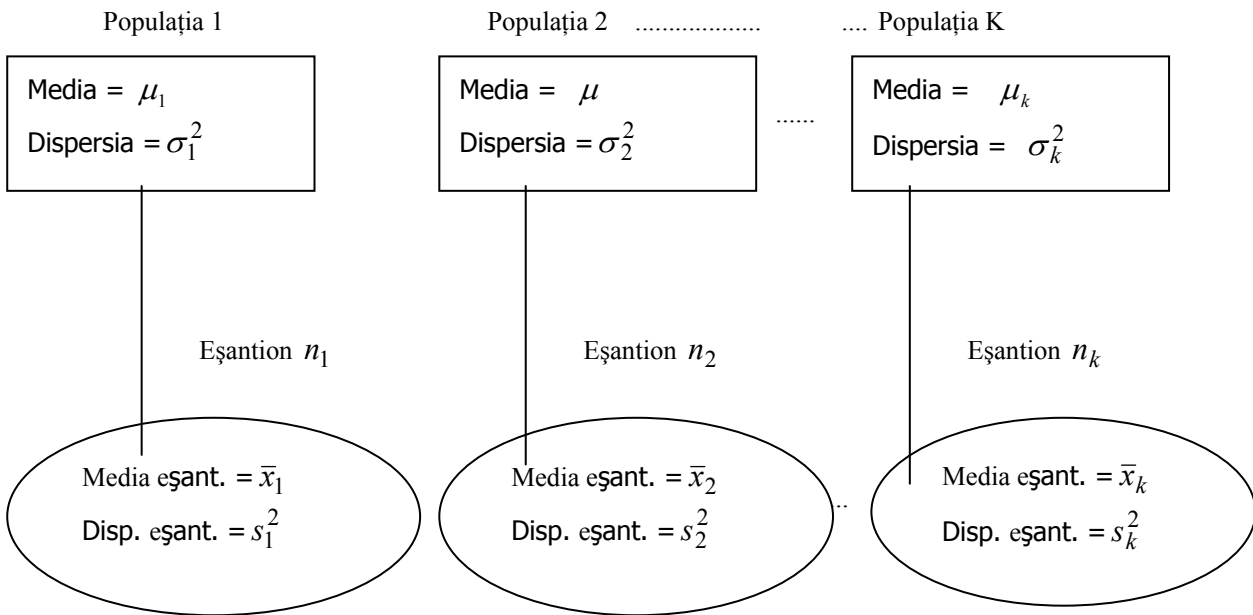
Populația 1   Populația 2 .... Populația K

| Media = μ_1 Dispersia = σ_1^2 | Media = μ Dispersia = σ_2^2 | Media = μ_k Dispersia = σ_k^2 |

......

Eșantion n_1 Eșantion n_2 Eșantion n_k

(Media eșant. $= \bar{x}_1$ Disp. eșant. $= s_1^2$)   (Media eșant. $= \bar{x}_2$ Disp. eșant. $= s_2^2$)   (Media eșant. $= \bar{x}_k$ Disp. eșant. $= s_k^2$)

Figure 1. The scheme for independent samples

For each volum population N_1, N_2,......., N_k, the volum samples n_1, n_2, ..., n_k are extracted. The value of characteristic X, corresponding to each unit from samples can be presented in a tabel such as their processing for obtaining the specific indicators is easy to make by handly or modern design. Such model is presented in tabel 1.

The characteristic X average corresponding to each sample is calculated as follows:

$$\bar{x}_j = \frac{\sum_{j=1}^{n_j} x_{ij}}{n_j} \tag{1.1}$$

Tab.1. **The notices for variance analysis of a characteristic, on independent samples**

Specification	Sample processing number:			
	1	2	.........j	...........k
The values of	x_{11}	x_{12}	x_{1j}	x_{1k}
characteristic X	x_{21}	x_{22}	x_{2j}	x_{2k}
	x_{31}	x_{32}	x_{3j}	x_{3k}
	.	.	.	.
	.	.	.	.
	x_{n1}	x_{n2}	x_{nj}	x_{nk}
The volum of the sample	n_1	n_2	n_j	n_k
The samples average	$\bar{x}_1$	$\bar{x}_2$	$\bar{x}_j$	$\bar{x}_k$

The general average of the characteristic corresponding to the general sample n, ($n = n_1 + n_2 + ... + n_k$), is calculated as follows:

$$\bar{\bar{x}} = \frac{\sum\limits_{j=1}^{k}\sum\limits_{i=1}^{n_j} x_{ij}}{n} \qquad (1.2)$$

Or as an moderated arithmetic average corresponding to each sample, using the formula:

$$\bar{\bar{x}} = \frac{\sum\limits_{j=1}^{k} \bar{x}_j n_j}{n} \qquad (1.3)$$

One of the objectives of variance analysis at the level of these samples constitues the verifying of the assumption of null difference, that mention thatt the samples averages are equal, that is:

$$H_0: \bar{x}_1 = \bar{x}_2 = ... = \bar{x}_j .... = \bar{x}_k$$

For the verifying of this assumption it can be used the statistical test F. The calculated of F value is compared with the tabel value; in case in what the calculated value is higher, the assumption H_0 is rejected; if it is lower, the assumption H_0 is accepted.

For the calculation of F value the following stages are:

a) Determination of the sum of the average deviations squares corresponding to each sample ($\bar{x}_j$) from the general average of the samples ($\bar{\bar{x}}$), well-balanced with the samples volume (n_j).

The calculation formula is:

$$SST = \sum\limits_{j=1}^{k}\left(\bar{x}_j - \bar{\bar{x}}\right)^2 n_j \qquad (1.4.)$$

Where SST are the symbols of the denomination of respective indicators in English (Sum of Squares for treatments).

b) Determination of the sum of variants deviations squares x_{ij} from the average corresponding to each sample ($\bar{x}j$). In this scope the following formula is used:

$$SSE = \sum\limits_{j=1}^{k}\sum\limits_{i=1}^{nj}\left(x_{ij} - \bar{x}_j\right)^2 \qquad (1.5)$$

where SSE = sum of square for error

c) Determination of the average of tests average deviations square ($\bar{x}_j$) from the general average ($\bar{\bar{x}}$), using the formula:

$$MST = \frac{SST}{k-1} \qquad (1.6.)$$

where MST- Mean square for treatments.

    d)   determination of average of values of deviations square x_{ij} from the tests averages $\bar{x}_j$, using the formula:

$$MSE = \frac{SSE}{n-k} \qquad (1.7.)$$

where MSE – mean square for error.

    e)   calculation of statistical test of signification F with the relation:

$$F = \frac{MST}{MSE} \qquad (1.8.)$$

We observe that the methodology presented offers the possibility of analysing the variance at each sample's level, SSE, noticying the influence of the factors which action into the samples and analyse the test.

$$SS\ (Total) = SST + SSE \qquad (1.9)$$

or:

$$\sum_{j=1}^{k}\sum_{i=1}^{n_j}\left(x_{ij} - \bar{\bar{x}}\right)^2 = \sum_{j=1}^{k}\left(\bar{x}_j - \bar{\bar{x}}\right)^2 n_j + \sum_{j=1}^{k}\sum_{i=1}^{n_j}\left(x_{ij} - \bar{x}_j\right)^2 \qquad (1.10.)$$

In the tabel 2 is presented a sintesis of all these indicators.

Tab.2. For the analysis of unidimensional variance of the samples

Variance source	Freedom degree	Square Sum	Square average	Statistical factor
$\bar{x}_j - \bar{\bar{x}}$	k-1	SST	$MST = \dfrac{SST}{k-1}$	$F = \dfrac{MST}{MSE}$
$x_{ij} - \bar{x}_j$	n - k	SSE	$MSE = \dfrac{SSE}{n-k}$	
Total	n – 1	SS (Total)		

Analysis of test indicators and estimation of selection base parameters

    One of the main objectives of this paper represents the elaboration of an expert system, specialized for statistical data processing from the sanitary system. This objective was reached, SASM being already experimentated on a data base processed from a primar record of a sanitary

cabinet from Drobeta Turnu Severin that includes in present 1791 persons, who are the potential and effective patients of this cabinet.

We mention the fact that the programme is very flexible, offering the possibility to perform all the updating operations and interogation data base, making easy the applying of processing and analysis statistical methods.

It is well known the fact that in many diseases, the illness is determined of some causes that are found in the enviroment, in the comportament, or in the nourishment, etc.

After the efectuated studies it noticed that the smoking is one of the most important risk factors that associated with other factors affect mostly the human health.

Analysing the structural diagrams given by SASM programme, the following aspects will be resultated:

➢ From the total of 1791 persons from the data base, 519 persons were diagnisticated with one of the diseases mentioned in listing O. M. S that means that the morbidity is high enough

➢ From the 519 persons mostly were the HTA sickmen (29,67%), CICD (11,76%), CI (8,768%) and UDD (8,74%)

➢ From the 519 persons, 258 are smokers representing about 50%'and 261 are nonsmokers

➢ Çomparing the data about the corresponding frequency distribution of the two structures (smokers and nonsmokers) depending of disease it can easily be observed that at at least 4 diseases the smokers sick procentage is more higher than of at nonsmokers.

For a better evidence of the influence of the smoking upon the morbidity, a bidimensional distribution of the patients with mentioned diseases (see table 3.1) was made.

From all the diseases, the arterial hypertension (HTA) is more spreaded at the smokers (37,37%) than at the nonsmokers, where 21,75% from the patients are ill.

Sensible differences are met at the ischemical chardiopathy (CI) where 10,82% from the patients are smokers, while 6,63% are nonsmokers. At the TBC patients 4,38% are smokers and 1,33% are nonsmokers.

There are also diseases where the nonsmokers procentage is higher than smokers. In such case is DZ, where the nonosmokers procentage is 6,37% and the smokers procentage is 2,32%; CICD – 12,73% nonsmokers and 10,82% nonsmokers; UDD – 9,55% nonsmokers and 7,99% smokers. This can be explained that the respective diseases are also favoured in an enough high proportion by some other factors or only by themselves; in such cathegory are: stress, nourishment, heredity, work conditions, environment, etc.

Very sugestive are the distributions curves presented in picture 3 that permit the selection of those diseases that are dominated throuh the highly frequences met in the study of sick people.

Significant is the fact that HTA is separated by the other diseases having the mximum value at the smokers structure.

Tab. 3 The patients distributions depend of the diagnosticated disease and the comportamental status (smoker or nonsmoker)

- % -

Disease	Total smokers	Male	Female
AB	4.38	4.67	2.99
AEV	0.77	0.93	0
ANGOR	0.52	0.62	0
AVC	0.26	0.31	0
BPCO	0.77	0.93	0
BPN	0.26	0.26	0
CI	10.82	11.53	7.46
CICD	10.82	11.53	7.46
CIROZA	0.25	0.31	0
DZ	2.32	2.49	1.49
ETILISM	0.52	0.62	0
FIA	0.52	0.63	0
GLAUC	0.77	0.63	1.49
GUTA	0.52	0.63	0
HEP	1.03	1.25	0
HTA	37.37	33.64	55.22
IC	1.03	1.25	0
IM	0.77	0.93	0
IRC	0.52	0.62	0
NEO	4.9	5.61	1.49
NYHA	0.26	0.31	0
OB	4.38	4.67	2.99
PSIHO	1.29	0.93	2.99
RA	1.03	1.25	0
SCHIZO	0.26	0.31	0
SP	0	0	0
STER	0.77	0.31	2.99
TBC	4.38	2.8	11.94
UDD	7.99	9.35	1.49
UG	0.52	0.63	0
Total	100	100	100

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Some Computational Aspects of the Symmetric Eigenproblem

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Abstract

In this paper we study some special computational aspects of the selfadjoint eigenproblem. We propose a new treatment of known numerical methods to compute eigenvalues and eigenvectors with high relative accuracy.
Theory, an error analysis and numerical experiments are presented.

Key words: elliptic eigenproblem, Rayleigh-Ritz method, Courant elements, multigrid, prolongation, high relative accuracy.

1   Introduction

The study of the eigenproblem has been very important in pure mathematics while the computation of the eigenvalues and eigenvectors of a matrix is necessary in the various fields of applied mathematics.

It is known that in mathematical economics in the input-output model the eigenvalue is a measure of the overall input-level.

In this paper we treat the possibilities of the improving the accuracy of numerical methods which are used in eigen-computation to obtain eigenvalues and eigenvectors with high relative accuracy. For this reason we choose the problem of the computation of the eigenvalues of selfadjoint problems.

In this paper we determine the global minimum (maximum) of the Rayleigh quotient with the preconditionated gradient method (1984).

To separate the appropiate eigenvectors we use the Rayleigh-Ritz method.

We use the multi-grid method in the sense of transfering the eigenvectors obtained on each grid to the finest grid and with that we start the next iteration.

The multi-grid method permits the computation of a selected eigenvalue and eigenvector.

We use finite element and finite difference discretization. In the case of the finite element discretization we assume that the family of triangulations is regular.

Here, the term "regular" means that exists a constant σ such that

$$\forall K \in \bigcup_h \mathcal{T}_h, \;\; \frac{h_k}{\rho_k} \leq \sigma$$

and the quantity $h = \max_{K \in \mathcal{T}_h} h_K$ approaches zero, where $h_K = diam(K)$,
$\rho_K = \sup\{diam(S); S$ is a circle contained in $K\}$.

For the eigenvalue approximation we refer to Babuška & Osborne [1] and Chatelin.

For the convergency theorem of the multi-grid method for the self-adjoint eigenvalue problem we refer to Hackbusch [2].

All numerical experiments in this paper have been done with the 2nd order elliptic eigenvalue problem.

We present a comparative study concerning on the effect - of the order of the used prolongation operators, the used discretization type and algebraic structure of the global stiffness matrix - to the accuracy.

2   Preliminary Results

In this paper the order of the prolongation operator we use in Hackbusch sense.

Definition 2.1 *[2] p is an interpolation of order m_p if interpolates polynomials of degree $m_p - 1$ exactly.*

In [4] we gave an approach method to realise a higher order interpolation in two dimension and we proved that interpolation of this type can be used as prolongation operator in multi-grid method.

The linear prolongation operator used generally in multi-grid method is of the second order.

In the following we present the main results from [4].

Theorem 2.2 *The polynomial interpolation formula on nine points has the following form:*

$$
\begin{aligned}
P_1(x,y) \;=\;& u_{i-1,j-1} + h[u_{\bar{x},ij-1}x + u_{\bar{y},i-1,j}y] + \\
&+ \frac{h^2}{2}[u_{\bar{x}x,ij-1} \cdot x(x-1) + u_{\bar{y}y,i-1j}y(y-1) + 2u_{\bar{x}\bar{y},ij}xy] + \\
&+ \frac{h^3}{2}[u_{\bar{x}x\bar{y},ij} \cdot xy(x-1) + u_{\bar{y}y\bar{x},ij}xy(y-1)] + \\
&+ \frac{h^4}{4}u_{\bar{y}y\bar{x}x,ij}xy(x-1)(y-1)
\end{aligned}
$$

Theorem 2.3 *If $u \in C^k(\bar{\Omega})$, where $k \geq 2$ integer then the interpolation polynom has a global error order of $\mathcal{O}(h^2)$.*

Thus

$$
P_1(x,y) = u((i-1)h,(j-1)h) + \frac{h}{1!}\left[\frac{\partial u(ih,(j-1)h)}{\partial x} \cdot x + \frac{\partial u((i-1)h,jh)}{\partial y}y\right] + \mathcal{O}(h^2).
$$

Theorem 2.4 *The polynomial interpolation formula on sixteen points has the following*

form:

$$P_2(x, y) = u_0 + (u_1 - u_0)x + \frac{u_2 - 2u_1 + u_0}{2}x(x-1) + (u_3 - 3u_2 + 3u_5 - u_0)$$

$$x(x-1)(x-2) + (u_4 - u_0)y + (u_5 - u_4 - u_1 + u_0)xy +$$

$$+\frac{u_6 - 2u_5 + u_4 - (u_2 - 2u_1 + u_0)}{2}xy(x-1) +$$

$$+\frac{u_7 - 3u_6 + 3u_5 - u_4 - u_3 + 3u_2 - 3u_1 + u_0}{6}xy(x-1)(x-2) +$$

$$+\frac{u_8 - 2u_4 + u_0}{2}y(y-1) + \frac{u_9 - 2u_5 + u_1 - (u_8 - 2u_4 + u_0)}{2}xy(y-1) +$$

$$+\frac{(u_{10} - 2u_9 + u_8) - 2(u_6 + u_4 - 2u_5) + (u_0 + u_2 - 2u_1)}{4}xy(x-1)(y-1)+$$

$$+\left[\frac{u_{11} - 3u_{10} + 3u_9 - u_8 - 2(u_7 - 3u_6) + 2(u_4 - 3u_5) + u_3 - 3u_2 + 3u_1 - u_0}{12}\right] \cdot$$

$$\cdot x(x-1)(x-2)y(y-1) + \frac{u_{12} - 3u_8 + 3u_4 - u_0}{6} \cdot y(y-1)(y-2) +$$

$$+\frac{u_{13} - 3u_9 - u_{12} + 3u_8 + 3u_5 - u_1 - 3u_4 + u_0}{6} \cdot xy(y-1)(y-2) +$$

$$+\left[\frac{u_{14} - 3u_{10} - 2(u_{13} - 3u_9) + u_{12} - 3u_8 + 3u_6 - u_2 + 2(u_1 - 3u_5) - u_0 + 3u_4}{12}\right] \cdot$$

$$\cdot x(x-1)y(y-1)(y-2) + \left[\frac{u_{15} - 3u_{14} + 3u_{13} - u_{12} + 3(3u_{10} - u_{11} + 3(u_8 - 3u_9)}{36}+\right.$$

$$\left.+\frac{3(u_7 - 3u_6) + 3(3u_5 - u_4) - u_3 + 3u_2 - 3u_1 + u_0}{36}\right] x(x-1)(x-2)y(y-1)(y-2).$$

Theorem 2.5 *Let $\Omega = (0,1) \times (0,1)$ with the boundary $\partial\Omega = \{0,1\} \times \{0,1\}$ and $u : \bar{\Omega} \to$ $\mathbf{R}$ a sufficiently smooth function, $u \in C^k(\bar{\Omega})$, $k \geq 2$ then the interpolation operator P_2 is of the third order.*

Thus we can write:

$$
\begin{aligned}
P_2(x, y) =\ & u((i-1)h,\ (j-1)h) + \\
& +\frac{h}{1!}\left[\frac{\partial u(ih,\ (j-1)h)}{\partial x} \cdot x + \frac{\partial u((i-1)h,\ jh)}{\partial y} \cdot y\right] + \\
& +\frac{h^2}{2!}\left[C_2^0 \frac{\partial^2 u(ih,(j-1)h)}{\partial x^2} \cdot x(x-1) + C_2^1 \frac{\partial^2 u(ih, jh)}{\partial x \partial y} \cdot x \cdot y +\right. \\
& +\left. C_2^2 \frac{\partial^2 u((i-1)h, jh)}{\partial y^2} \cdot y(y-1)\right] + \frac{h^3}{3!}\left[C_3^0 \frac{\partial^3 u(ih, (j-1)h)}{\partial x^3} x(x-1)(x-2) +\right. \\
& +C_3^1 \frac{\partial^3 u(ih, jh)}{\partial x^2 \partial y} \cdot xy(x-1) + +C_3^2 \frac{\partial^3 u(ih, jh)}{\partial y^2 \partial x} \cdot xy(y-1) + \\
& +C_3^3 \frac{\partial^3 u((i-1)h, jh)}{\partial y^3}y(y-1)(y-2)\right] + \mathcal{O}(h^3).
\end{aligned}
$$

The obtained theoretical results were tested numerically. For this reason we have written a program in MATLAB.

For tests we consider functions defined on $\Omega = (0,1) \times (0,1)$, which on the boundary of the domain denoted by $\Gamma = \{0,1\} \times \{0,1\}$, verify the condition $u = 0$.

We studied the order of the error in connection with the number of gridpoints on $\Omega_h = \{(ih, jh) \,|\, i,j = \overline{0,N}\}$ where $h = \frac{1}{N}$ is the meshsize and $N > 0$ is a given integer number.

With the increasing of the number of meshpoints the error decreased semnificatively.

The obtained error orders in both cases we can see in the following tables, where N means the number of the grid points. The choise of N is unlimitated, depends only on the configuration of the computer.

N	error order	N	error order
4	10^{-3}	4	10^{-3}
16	10^{-4}	16	10^{-5}
32	10^{-5}	32	10^{-6}
64	10^{-6}	64	10^{-8}
128	10^{-7}	128	10^{-9}
256	10^{-8}		

3 Numerical results

Test Problem

All numerical experiments have been done with the computation of the eigenvalues and eigenvectors of the Laplace-operator on the unit square, with first order boundary conditions on the boundary of the domain.

$$\begin{cases} \Delta u + \lambda u = 0, & x \in \Omega = (0,1) \times (0,1) \\ u = 0, & x \in \partial\Omega \end{cases} \tag{3.1}$$

We can determine exactly the eigenvalues of the Laplace operator.

It's known that the exact values are: $\lambda_1 = 2\pi^2$, $\lambda_2 = \lambda_3 = 5\pi^2$ $\lambda_4 = 8\pi^2$.

We are interested in the approximation of the eigenpairs of the generalized eigenvalue problem.

The variational form of he considered problem is:

$$\begin{cases} u \in H_0^1(\Omega), & \lambda \in \mathbf{R} \\ a(u,v) = \lambda b(u,v), & \forall v \in H_0^1(\Omega) \end{cases} \tag{3.2}$$

where

$$a(u,v) = \sum_{i,j=1}^{2} \int_{\Omega} \frac{\partial u}{\partial x_i} \cdot \frac{\partial v}{\partial x_j} dx$$

and

$$b(u,v) = \int_{\Omega} uv dx.$$

$H_0^1(\Omega)$ denote the usual Sobolev space on Ω.

We are interested in the approximation of the generalized eigenpairs by multi-grid method and we denote by

$$Av = \lambda Bv \qquad (3.3)$$

the approximate eigenvalue problem, associated to the problem (3.2), where

$$A = A^T > 0, \quad B = B^T > 0, \quad A, B \in \mathcal{M}_n(\mathbf{R}).$$

The Rayleigh quotient we denote by:

$$\rho(x) := \frac{(Ax, x)}{(Bx, x)}.$$

The properties of $\rho(x)$ we can see in ([3]).

Numerical methods. Adaptive algorithm.

To determine the global minimum (maximum) of the quotient we use the preconditionated gradient method which we started with:

$$x_0 \text{ given}, \quad x_{m+1} = x_m - t_m(A - \rho_m B)x_m, \ m = 0, 1, 2, \ldots$$
$$\rho_m := \rho(x_m). \qquad (3.4)$$

t_m is a real positive number choosen as to have:

$$\rho(x(t)) < \rho(x_m), \qquad (3.5)$$

where

$$x(t) := x_m - t d_m, \quad d_m := (A - \rho_m B)x_m, \quad \|x_m\|_B = 1.$$

Algorithm

Let $\quad k - 1 \geq 0$, the number of the eigenvectors, $v^{(i)}$, what we want to calculate

$\varepsilon > 0$, precision

max, the number of the iteration

x_0, the first approximation of the kth eigenvector

1. for $m := 0, \max$
2. $x_m := x_m - \sum_{i=1}^{k-1}(B\, x_m, v^{(i)}) \cdot v(i)$
3. $r := (B\, x_m, x_m)$

   if $r \leq 0$ then choose another approximation for x_m and go to 1.

   else continue
4. $\rho_m = \frac{(A\, x_m, x_m)}{r}$
5. $r := \sqrt{r}, \quad x_m := \frac{x_m}{r}$
6. $d_m := P^{-1}(A - \rho_m B)\, x_m$
7. $d_m := d_m - \sum_{i=1}^{k-1}(B\, d_m, v^{(i)}) \cdot v^{(i)}$
8. $bd := (B\, d_m, d_m)$

   if $bd \leq \varepsilon$ then go to 11

   else continue
9. $t_m := tv(x_m, d_m, \rho_m, bd)$
10. $x_{m+1} := x_m - t_m * d_m$
11. $v^{(k)} := x_{m+1}/(B\, x_{m+1}, x_{m+1})^{1/2}$

$\lambda_k := \rho_m$

The proccedure for the choise of t:

1. $c := (d_m, (A - \rho_m B)x_m), ad := (A\, d_m, d_m), rd := ad/bd.$

2. $b := bd * (rd - \rho_m), bx := (B\, d_m, x_m), a := b * bx - c * bd$

3. if $b < \min(0, c * (3bx + (bd)^{1/2})$

$$\text{then } tv := 1/(bd)^{1/2}$$
$$\text{else } tv := 2c/(b + \sqrt{b^2 - 4ac})$$

The convergency of the method was proved in 1984 by V.G. Prikazcsikov, A.N. Himics.

For the precondition matrix we will use in the following $P = \text{tridiag} (-1, 4, -1)$.

To obtain our numerical results, we have written a program in MATLAB. The experiments were done using a PC with AMD K7 ATHLON 750 Mhz processor and 512 MB memory.

It is known that the approximate values are:$\lambda_1 = 2\pi^2 \approx 19,74$ and $\lambda_2 = 5\pi^2 \approx 49,35$.

At first we use the multi-grid method with finite element discretization (MGFED).

The eigenvalues obtained by applying the MGFED approximate the eigenvalues of the differential operator from above.

For our experiment we choose prolongation operators of different order.

The interpolation operator denoted by P_1 is of the third order and the interpolation operator denoted by P_2 is of the fourth order. The linear prolongation operator, generally used in the multi-grid method, is of the second order and we'll denote it by P.

Figure 1 and 2 present the obtained maximal errors on logarithmic scale in connection with the used prolongation operator when we calculated the first two approximate eigenvalue, on $2, 3, 4, 5$ grids.

We used 2 interior, 2 exterior iteration and we started the iterations with 5 gridpoints on the first grid (on the 5th grid we got 16129 gridpoints).

	1	2	3	4
P	6,48E-06	2,0540E-07	1,051E-08	6,17E-10
P1	4,37E-06	1,095E-07	4,89E-09	2,52E-10
P2	4,54E-06	1,046E-07	4,95E-09	2,85E-10

Figure 1

	1	2	3	4
P	1,25E-05	4,3160E-07	2,351E-08	1,409E-09
P1	1,0650E-05	2,493E-07	1,1630E-08	6,8560E-10
P2	1,2540E-05	4,159E-07	2,22E-08	1,3160E-09
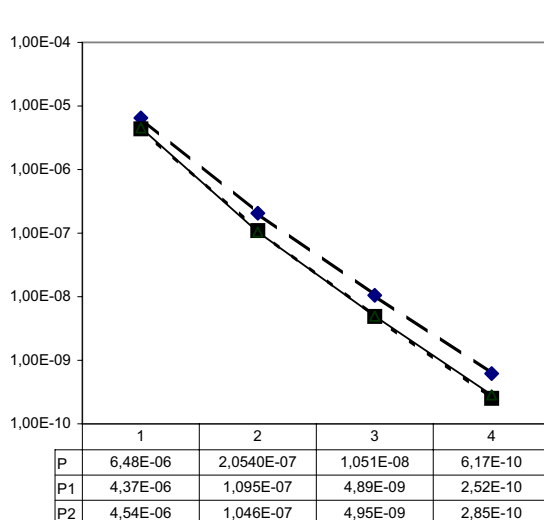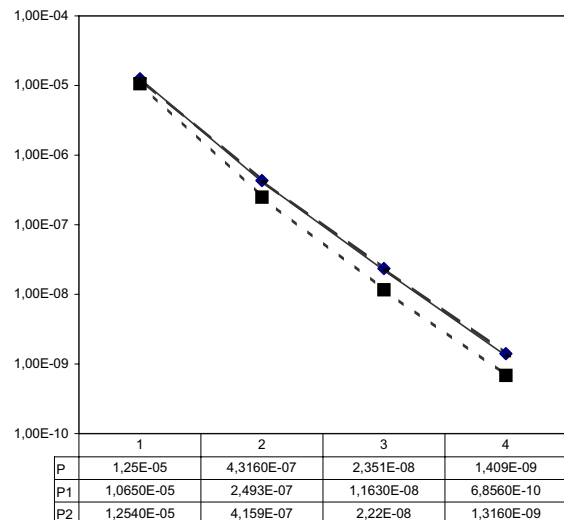
Figure 2

We can observe on the figures 3 and 4 that we can improve the accuracy semnificatively if we increase the number if the interior and exterior iterations.

We can see the maximal errors obtained on $2, 3, 4, 5$ grids with 8 exterior and 4 interior iterations. We used 5 gridpoints also, on the first grid.
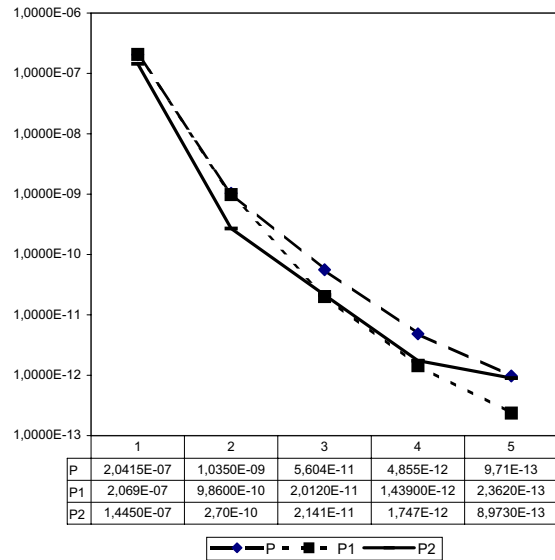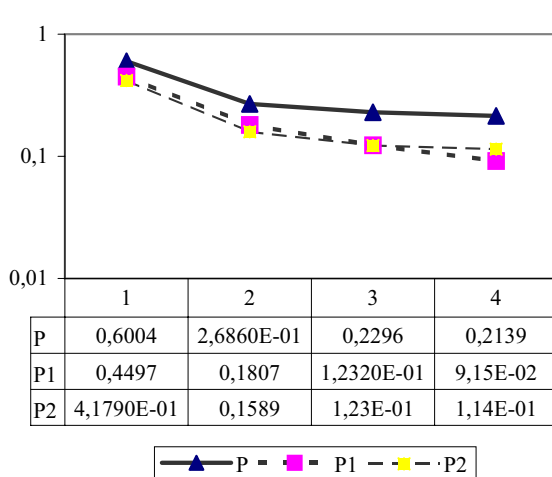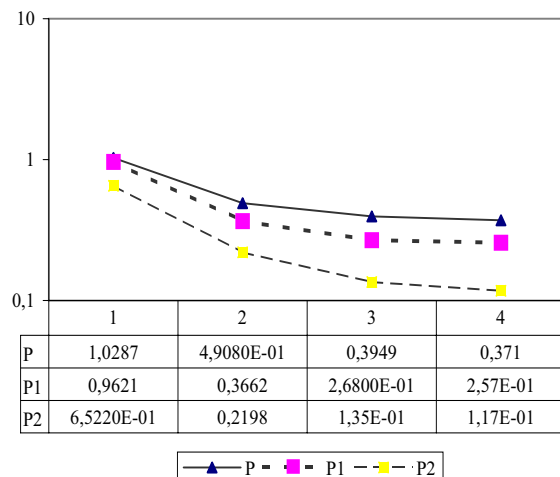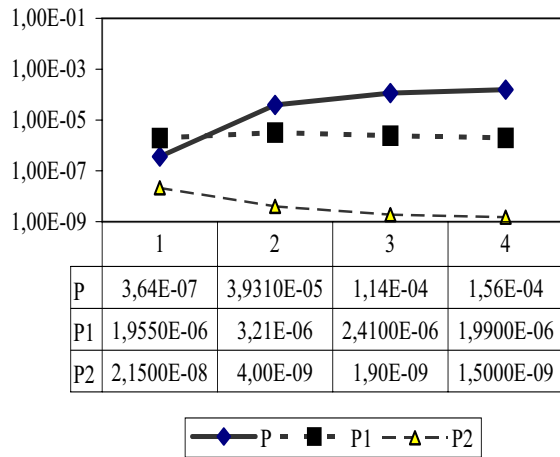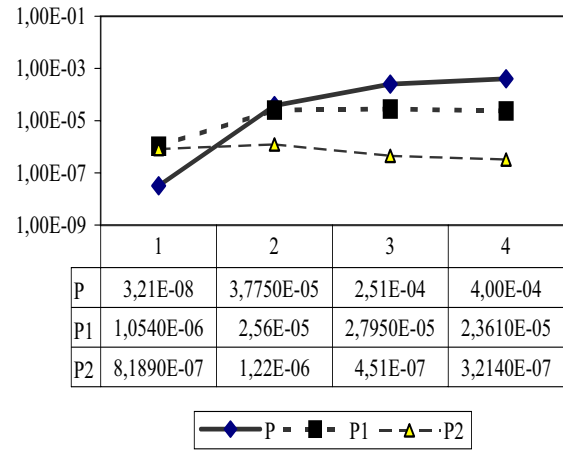
	1	2	3	4	5
P	8,3000E-12	6,8000E-11	1,2590E-11	1,0690E-12	6,3340E-13
P1	2,06E-12	1,0300E-11	1,6400E-12	2,98300E-	3,7560E-13
P2	1,1069E-12	8,40E-12	1,82E-12	3,11E-13	1,7470E-12

Figure 3
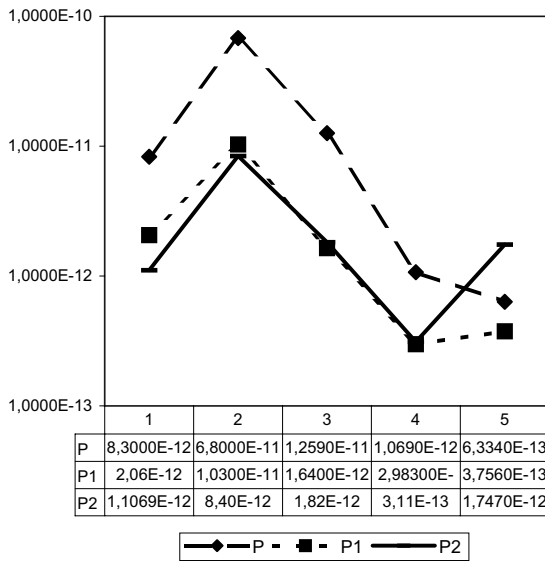
	1	2	3	4	5
P	2,0415E-07	1,0350E-09	5,604E-11	4,855E-12	9,71E-13
P1	2,069E-07	9,8600E-10	2,0120E-11	1,43900E-12	2,3620E-13
P2	1,4450E-07	2,70E-10	2,141E-11	1,747E-12	8,9730E-13

Figure 4

To study the effect of the used discretization type to the accuracy in the following we present the maximal errors obtained with the same numerical method, in the same conditions, but with finite difference discretization (MGFDD).

The eigenvalues obtained by applying the MGFDD approximate the eigenvalues of the differential operator from below.

Figure 5 and 6 present the obtained maximal errors on logarithmic scale in connection with the used prolongation operator order when we calculated the first two approximate eigenvalue on $2, 3, 4, 5$ grid.

We used 2 interior, 2 exterior iteration and we started the iterations with 5 gridpoints on the first grid (on the 5th grid we got 16129 gridpoints).

	1	2	3	4
P	0,6004	2,6860E-01	0,2296	0,2139
P1	0,4497	0,1807	1,2320E-01	9,15E-02
P2	4,1790E-01	0,1589	1,23E-01	1,14E-01

Figure 5

	1	2	3	4
P	1,0287	4,9080E-01	0,3949	0,371
P1	0,9621	0,3662	2,6800E-01	2,57E-01
P2	6,5220E-01	0,2198	1,35E-01	1,17E-01

Figure 6

We present in the following on figure 7 and 8 the obtained maximal errors on $2, 3, 4, 5$ grid with 8 exterior and 4 interior iterations. We used 5 gridpoints also, on the first grid.

	1	2	3	4
P	3,64E-07	3,9310E-05	1,14E-04	1,56E-04
P1	1,9550E-06	3,21E-06	2,4100E-06	1,9900E-06
P2	2,1500E-08	4,00E-09	1,90E-09	1,5000E-09

Figure 7

	1	2	3	4
P	3,21E-08	3,7750E-05	2,51E-04	4,00E-04
P1	1,0540E-06	2,56E-05	2,7950E-05	2,3610E-05
P2	8,1890E-07	1,22E-06	4,51E-07	3,2140E-07

Figure 8

If we compare the effect of the discretization type to the accuracy, as we can see on figure 9, we can deduce that the finite element discretization is more efficient. For numerical test we used P_2 as prolongation operator and we calculate the approximate eigenvalues on $2, 3, 4, 5$ grid with 8 exterior and 4 interior iterations. We started with 5 gridpoints the iterations on the first grid.

Maximal errors on logarithmic scale in connection with the number of grids using different type of discretizations

	1	2	3	4
FED	6,48E-06	2,0540E-07	1,051E-08	6,17E-10
FDD	6,00E-01	0,2686	2,30E-01	2,14E-01

Figure 9

Maximal errors on logarithmic scale in connection with the used prolongation operator (FED)

	1	2	3	4
PTR	2,0540E-07	1,051E-08	6,17E-10	3,78E-11
PI	5,52E-06	2,84E-07	1,67E-08	1,00E-10
PPT	2,06E-06	5,76E-07	1,23E-07	2,70E-08

Figure 10

On figure 10 we can analyse the effect of the precondition matrix to the accuracy. In the presented table PTR means pentadiagonal preconditioner, PI means that we use only the gradient method and PPT presents the tridiagonal precondition matrix.

We studied the accuracy of the eig proccedure (EP) in comparation with the efficienty of the preconditionated gradient method (PGM). The computational time in seconds are presented also.

N_h	Methods	λ_1	λ_2	$e_{\max}$		Time (sec)
4	EP	21,7222	57,7975	$0,0315 \cdot 10^{-11}$	$0,4668 \cdot 10^{-11}$	0
	PGM					0,2800
9	EP	20,2284	52,0803	$0,0007 \cdot 10^{-9}$	$0,6689 \cdot 10^{-9}$	0,1600
	PGM					0,4900
19	EP	19,8626	50,0413	$0,1582 \cdot 10^{-10}$	$0,1131 \cdot 10^{-10}$	44,2100
	PGM					45,0900

Figure 11 presents the obtained maximal errors when we used PGM in comparation with the PGM combinated with the multi-grid method using 3, 4, 5 grids. The number of interior and exterior iteration is 2, and we started with 5 gridpoints on the first grid.

Maximal errors on logarithmic scale
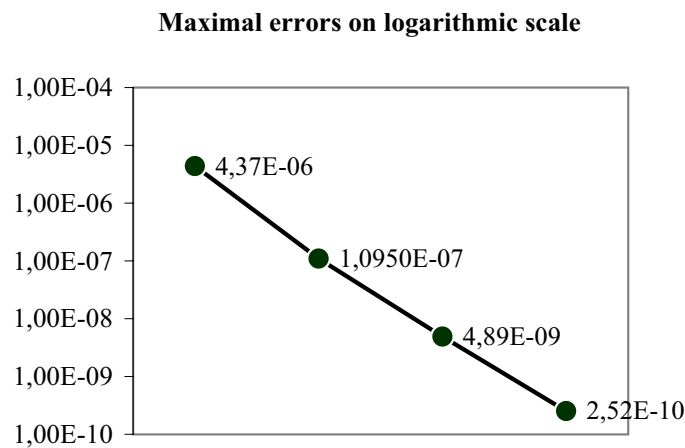
Figure 11

4   Conclusions

Using the numerical results from Section 3 we get information concerning the connection between the degree of the used prolongation operator, the used discretization type and the accuracy of the used method.

Error analysis and numerical experiments suggested that the accuracy of multi-grid method strongly depends on the used prolongation operator order.

We use the the multi-grid method to increase the accuracy in the sense of transfering the eigenvectors obtained on each grid to the finest grid and with this we start the next iteration.

We can observe on the presented figures that if we use prolongation with high accuracy to transfer the eigenvectors obtained on each grid to the finest grid, then the decrease of the accuracy is minimal and the iteration on the finest grid will start better.

Thus using higher order prolongation operator we can increase the accuracy.

Numerical tests show that we can improve the accuracy if we increase the number of grids and the number of interior and exterior iterations.

If we use third order prolongation than the accuracy will increase semnificatively in comparation with the second order prolongation, but with the fourth order prolongation the increasing it's innesential.

We can observe that the used precondition matrix has a semnificative influence to the accuracy and the tridiagonal preconditioner is the most efficient in the case of the finite element discretization.

We can study the convergence of the eigenvectors.

The suitable eigenfunctions from the space $V_h = \{w_1, \ldots, w_n\}$ are:

$$u_h^{(i)} = \sum_{k=1}^{n} v_k^{(i)} \cdot w_k, \quad i = \overline{1, n}$$

The program permits the calculation of more than two eigenvalues.

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An Application of Groebner Baseses

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Abstract

The aim of this paper is to investigate the conditions under a given matrix has a particular Jordan structure. Due to computational complexity we will limit ourself to matrices of order 2 and 3, giving explicit conditions, while our method conceptually works in any dimension.

Let us suppose we have a square matrix A with complex entries. Then its Jordan form exists, and it is a matrix having Jordan cells on its diagonal, each cell has one out of all proper values of A on the diagonal positions, and 1-s upper (or if we like, below) the diagonal.

We ask the following quite natural question: How can we characterize all the matrices having a specific Jordan structure? More precisely, what conditions have to be satisfied by the matrix entries to guarantee a given specific Jordan structure? So we will examine the next problem:

Problem. *Suppose given a matrix with complex entries such as*

$$\begin{pmatrix} x & y \\ z & v \end{pmatrix} \ respectively \ \begin{pmatrix} x & y & z \\ r & s & t \\ u & v & w \end{pmatrix}.$$

Try to characterize the matrix's entries, if it is known that its Jordan form is

$$\begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix} \ respectively \ \begin{pmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix} \ or \ \begin{pmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{pmatrix}.$$

We apply Groebner bases to solve these problems. The main idea is as follows. We express A in a general way using its Jordan form. The entries of the base-changing matrices, as well as λ can be viewed as parameters in the resulting polynomial system of equations. Using elimination theory we obtain a set of polynomial relations between the entries of the given matrices, which is the set of relations we were looking for.

Al these computations can be made (only) using an appropriate computer algebra system. We used the free package *Singular* [], which is one of the most powerful packages performing Groebner bases algorithms.

To give the details, our results are summarized in the following theorems.

Theorem. *Let us consider the matrices*

$$A = \begin{pmatrix} x & y \\ z & v \end{pmatrix} \ and \ D = \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}.$$

If D is the Jordan form of A then

$$(x - v)^2 + 4yz = 0.$$

Proof. Let us express

$$A = CDC^{-1}, \tag{1}$$

where C is a nonsingular matrix, corresponding to the appropriate base change. Let us denote

$$C = \begin{pmatrix} a & b \\ c & d \end{pmatrix}.$$

We can suppose $\det(C) = 1$, (this will be actually the first equation) otherwise we can divide all the entries of C by a square root of $\det(C)$ (not zero) and we obtain a C making the job and having determinant 1. Now (1) gives a system of four polynomial equations which express the entries of A as functions of the parameters – the entries of C and D.

To suppress the parameters we wrote the following program in Singular.

```
ring R=0,(a,b,c,d,l,x,y,z,v),(dp(5),dp(4));
matrix  A[2][2]   = x, y, z, v;    // A
matrix  D[2][2]   = l, 1, 0, l;    // D
matrix  C[2][2]   = a, b, c, d;    // C
matrix C1[2][2]   = d,-b,-c, a;    // det(C)*C^{-1}
matrix  B[2][2]   = C*D*C1;        // B = C * D * C^-1 * det(C)
matrix  M = A - B;
//
// The system of the 1 + 4 equations
//
ideal i=ad-bc-1,M[1,1],M[1,2],M[2,1],M[2,2];
i;
```

We obtained the following "system" of 5 equations:

```
i[1]=-bc+ad-1
i[2]=bcl-adl+ac+x
i[3]=-a2+y
i[4]=c2+z
i[5]=bcl-adl-ac+v
```

and by elimination we get:

```
//
// Elimination of (a,b,c,d,lambda)
//
ideal j=eliminate(i,abcdl);
j;
```

$$x^2 + 4yz - 2xv + v^2 = 0,$$

which is just the relation claimed in the theorem.

Let us now examine the case of dimension 3. We have the following result.

Theorem.   *Let us now consider the matrices*

$$A = \begin{pmatrix} x & y & z \\ r & s & t \\ u & v & w \end{pmatrix} \ and \ D = \begin{pmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{pmatrix}.$$

If D is the Jordan form of A then

$$
\begin{aligned}
3yu - 2xv + sv + vw &= 0 \\
xu - 2su + 3rv + uw &= 0 \\
xs + s^2 + 3zu + 6tv - xw - 3sw + 2w^2 &= 0 \\
3zr - 2xt + st + tw &= 0 \\
3yr - 2xs + s^2 - 3zu + 2xw - w^2 &= 0 \\
xr + rs + 3tu - 2rw &= 0 \\
xz - 2zs + 3yt + zw &= 0 \\
xy + ys + 3zv - 2yw &= 0 \\
2x^2 + 3yr - xs + 6zu - 3xw + sw + w^2 &= 0 \\
3s^2u + 3zu^2 - 2xrv - 5rsv - 2xuw - 2suw + rvw + uw^2 &= 0 \\
rsu + tu^2 - r^2v - ruw &= 0 \\
3zs^2 - 2xyt - 5yst + 3z^2u - 2xzw - 2zsw + ytw + zw^2 &= 0 \\
yzs - y^2t + z^2v - yzw &= 0.
\end{aligned}
$$

Proof.   Let us now express C as the matrices

$$C = \begin{pmatrix} a & b & c \\ f & g & h \\ k & m & n \end{pmatrix},$$

and suppose also $A = CDC^{-1}$ as in (1). We then obtain o system of 11 equations, which can be produced also by a Singular program. Here it is this program:

```
ring R=0,(a,b,c,f,g,h,k,m,n,l,x,y,z,r,s,t,u,v,w),(dp(10),dp(9));
matrix A[3][3]  = x,y,z,r,s,t,u,v,w;     // A
matrix D[3][3]  = l,1,0,0,l,0,0,0,l;     // D
matrix C[3][3]  = a,b,c,f,g,h,k,m,n;     // C
matrix C1[3][3] = gn-hm,cm-bn,bh-cg,     // C^{-1}*det(C)
                  hk-fn,an-ck,cf-ah,
                  fm-gk,bk-am,ag-bf;
matrix B[3][3] = C*D*C1;                 // B = C*D*C^-1 * det(C)
matrix M[3][3] = A - B;
ideal i= agn+cfm+bhk-cgk-ahm-bfn-1,
 M[1,1],M[1,2],M[1,3],M[2,1],M[2,2],M[2,3],M[3,1],M[3,2],M[3,3];
i;
ideal j=eliminate(i,abcfghkmnl);
j;
```

Now the system of parametric equation of entries for A (expressed as the ideal generators for i) is

```
i[1]=-cgk+bhk+cfm-ahm-bfn+agn-1
i[2]=cgkl-bhkl-cfml+ahml+bfnl-agnl-ahk+afn+x
i[3]=ack-a2n+y
i[4]=-acf+a2h+z
i[5]=-fhk+f2n+r
i[6]=cgkl-bhkl-cfml+ahml+bfnl-agnl+cfk-afn+s
i[7]=-cf2+afh+t
i[8]=-hk2+fkn+u
i[9]=ck2-akn+v
i[10]=cgkl-bhkl-cfml+ahml+bfnl-agnl-cfk+ahk+w
```

and the elimination gives j.

```
j[1]=3yu-2xv+sv+vw
j[2]=xu-2su+3rv+uw
j[3]=xs+s2+3zu+6tv-xw-3sw+2w2
j[4]=3zr-2xt+st+tw
j[5]=3yr-2xs+s2-3zu+2xw-w2
j[6]=xr+rs+3tu-2rw
j[7]=xz-2zs+3yt+zw
j[8]=xy+ys+3zv-2yw
j[9]=2x2+3yr-xs+6zu-3xw+sw+w2
j[10]=3s2u+3zu2-2xrv-5rsv-2xuw-2suw+rvw+uw2
j[11]=rsu+tu2-r2v-ruw
j[12]=3zs2-2xyt-5yst+3z2u-2xzw-2zsw+ytw+zw2
j[13]=yzs-y2t+z2v-yzw
```

These are actually the conditions we were looking for.

Finally let us include the second variant we examined of the Jordan form. The theorem is similar.

Theorem. *Let us now consider the matrices*

$$A = \begin{pmatrix} x & y & z \\ r & s & t \\ u & v & w \end{pmatrix} \ and \ D = \begin{pmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{pmatrix}.$$

If D is the Jordan form of A then

$$
\begin{aligned}
x^2 + 3yr - xs + s^2 + 3zu + 3tv - xw - sw + w^2 &= 0 \\
3xyr + 11x^2s + 45yrs - 17xs^2 + 14s^3 + & \\
3xzu + 18zsu + 27ytu + 27zrv - 24xtv + & \\
45stv - 7x^2w - 36yrw + 8xsw - 21s^2w - & \\
9zuw - 9tvw + xw^2 + 15sw^2 - 4w^3 &= 0 \\
9y^2r^2 - 11x^3s - 48xyrs + 17x^2s^2 + & \\
3yrs^2 - 14xs^3 - 3x^2zu + 9yzru - & \\
18xzsu - 27xytu - 27xzrv + 24x^2tv + & \\
9yrtv - 45xstv + 7x^3w + 33xyrw - & \\
8x^2sw - 3yrsw + 21xs^2w + 9xzuw + & \\
9xtvw - x^2w^2 + 3yrw^2 - 15xsw^2 + 4xw^3 &= 0.
\end{aligned}
$$

The proof is essentially the same as for the previous theorem, so we leave it to the reader as a computer algebra exercise.

Let us make some final comments. As it can be noticed the theorems above compute necessary conditions for a matrix to exhibit a special Jordan form. It is a question – planned to be examined in a subsequent work – how these conditions can be refined to sufficient ones, if they can be at all.

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MEDICAL STATISTIC ANALYSIS SYSTEM

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Abstract: *SAMS PROVIDES conclusions regarding the influence of smoking on medical affections. Mainly, SASM allows for: the calculus of probabilitis distribution in any of the 30 medical affections; the calculus of the probabilities distribution of the medical affections in function of the patient job; finding out the necessary information about the most frequent affection related to job and to the feature smoker/non-smoker; learning the smoking illness risk in conection with job groups.*

Taking into consideration the way the SAMS has been designed and implemented (using the elements of programing-object in Java language and the Prolog language), the extention of the study to the influence of other factors such as alcohol or drug abuse on the health state could be easily realized.

1. General presentation

Medical statistic analysis system, uses the data regarding the diseases of 1791 patients from Drobeta Turnu Severin. The information regarding these patients include:

- Occupation
- 30 possible diseases
- patient identification (name, personal numeric code, address)

The utilized **technology** in impementation of the application is a combined technology SUN-MICROSOFT. Precisely, the MSAS aplication uses:

- Java language;

- PROLOG language – implemented in JAVA and interface JIPROLOG version 2.0 fot comunication JAVA-PROLOG realised by Ugo Chirico (http://www.ugosweb.com);
- Java Data Base Connectivity and JDBC-ODBC bridge communication for data base;
- Microsoft Excel.

The reasons for we used the above mentioned techologies are the following:

- The portability offered by the JAVA language, the independence from platform;
- The variety of graphic means offered by JAVA language and the its processing power;
- The simplity of using the data base from EXCEL (updaing the base, its visualisation, etc.);
- The extension possibility of interfaces engine through the capacity and the means offered by PROLOG language.

For the running SAMS the user does not necessary know the JAVA language because:

- The launching in running is realised under the operation system WINDOWS (98, 200 or XP) with one click on the application icon on the desktop;
- MSAS communicates with the user through the graphic interface which uses windows, buttons and menu pop-up;
- The SAMS data base updating is realised under EXCEL.

MSAS application presents the principal characteristics of an expert system:

- Contains a data base;
- Contains an interfaces engine that has the following base jobs:
    1. obtains the conclusions\
    2. follows the execution stages and depending of these it give the conclusions
- It is based on a graphic interface easy to use by user.

Mainly, MSAS supplies the conclusions regarding the influence of the smoking upon the medical diseases. It may be uses succesfully in many other scopes.

MSAS mainly permits:

- The calculation of the probabilities distribution on any disease group from all 30 diseases;
- The calculation of the probabilities distribution of medical diseases depending of patient occupation;
- Finding the necessary information regarding the most frequenty disease depending the occupation and characteristic smoker/not smoker;

- Finding the risk of getting ill in case of smoking, on occupation groups.

The conclusions variety offered by MSAS is big if we take into consideration all the possible combinations:

- Calculations for whole patients sample;
- Calculations for all diseases or only for a part of these, established by user;
- Calculations for all the occupations or only fof a part of these.

Taking into the account the way in what MSAS was designed and implemented, the using of programming elements, subject of JAVA language and using of PROLOG language, it isn't hard to realised the extension of the study regarding the influence of other factors upon the health (as: alchool consumtion, for which there are information data base).

The MSAS processing power is assured by the present technology used in implementation. JAVA-PROLOG communication grants a supplementary degree of modulation and access at the programming elements and superior extension degree.

Through the JAVA language specific, MSAS is a easy but strong program, that can run not only under WINDOWS but also under UNIX. It can be accessed from Internet by using WEB technics. Visual means addressed to the user are materialised in PIE and BARE graphics.

In conclusions, MSAS is an expert mini system, that in the present form gives conclusions regarding the influence upon the health.

The obtained conclusions are not universal valid because the performed study refers at the population from a certain geographic area, and the enviroment and the nourishment also influences the health.

2. The structure of MSAS system

In this paragraph we shall present the components of MSAS system and we shall give a short describe of these. MSAS has the following components, represented in **Picture 1:**

1. *A graphical User Interface* GUI (Grafhical User Interface).

The GUI interface is presented in **Picture 2** and contains:

- Pop-up menu for occupations selection;
- Pop-up menu for diseases selection;
- TextArea windows for:
    - display the diseases selected by user;
    - display the occupations selected by user;
    - display the conclusions obtained by MSAS;

- display the guide MSAS contained the main successions for USER.

- buttons for selection confirmation of the diseases and occupations;

- buttons for statistical processing;

- buttons for actions.

The GUI interface is launched through performing of a double click on MSAS icon of desktop.

2. *INIT component*

This component, implemented in JAVA and PROLOG initiates the MSAS system through the charging the disease and occupations lists.

The initialisation component permits the selection of all diseases from the data base without searching disease by disease. The way in that the user performed the selection leads to the display of some information regarding the diseases selection. The display is effectuated in the first of the four windows TextArea.

3. *Aquisition module A1*

Has the task to pick and memorise the user options in initialisation. In case when the user wants to drop the made selections instead of others, the button from GUI interface cancel the disease is pushed and the diseases selection process is replayed.

4. *Graphical component*

Performs the PIE and/or BARE graphics depend of the data given by Numerical Processing Module.

If we note $E=\{a_{10}, \ldots, a_{30}\}$, the lots of diseases from the data base then for any subcrowd $X \subseteq E$, selected with INIT and A1 components, MSAS considers:

- The distribution field CD (X, S, oc) of diseases from X diagnosticatedfor the smokers from the data base having the occupation oc;

- The distribution field CD (X, NS, oc) for nonsmokers;

- The CD (X, S, T) field for all smokers;

- The CD (X, S, T) field for all nonsmokers;

- The CD0 (X, T, oc) field associated to X, for all the patients having the occupation oc;

- The CD0 (X, T, T) field associated to X, for all the patients and all occupation.

In **Picture 3** we presented the PIE graphic for all the occupations (smokers and nonsmokers).The The BARE graphics present yhe most frequently disease for each occupation, for all the three category:

- Smokers;

- Nonsmokers;

- Smokers + nonsmokers.

In **Picture 4** we present the result of the request of PIE graphic for the smokers. It is noticed in this picture the fact that for each occupation the most frequently disease for the smokers is presented.

5. *Numerical processing module*

This module has the following tasks:

- Performs the connection MSAS with the data base;
- Extracts the necessary information from tha data base.

According with the specifications of INIT and A1 module, this module procceses the extracted data from the data base and transmits the result of proccesing of PIE and BARE graphics according with the user's specifications

6. *Conclusions module*

The tasks performed by this module are the following:

- Follow the tasks performed by MSAS according with the user's specifications, retains the results obtained and gives the conclusions regarding the risk of getting ill at the smokers;
- Perform of some symbolic processings necessary for the graphical representations.

7. *A2 module*

The main task of this module is taking over the conclusions by the Conclusions module and displaying them in the last window TextArea of the GUI interface.

8. *Data base*

The data base is a file named *Afectiuni.csv.*

It is created in JAVA with a program separate of MSAS. The MSAS system doesn't contain components for the base updating, because the introduction of such components would have complicated the interface structure of the MSAS system, and on the other hand the base updating can be made in Excel, independently of MSAS.

USER

Interfață grafică utilizator – GUI
- Meniuri po-up
- Butoane
- Ferestre TextArea

J

Componenta

INIT
- Inițializarea SASM

J+P

Modulul A1
- Achiziție de informații de la USER

J

Modulul CONCLUZII
- Procesări simbolice
- Motor pentru formularea concluziilor

J

Modulul A2
- Achiziție de cunoștințe pentru USER

J

Componenta Grafică
- Grafică PIE
- Grafică BARE

J

Modulul de Procesare Numerică
- Extragere din baza de date
- Procesări numerice

J

JDBC - ODBC bridge
EXCEL

Baza de date
Afecțiuni.csv

Fig. 1 Componentele SASM și interdependența acestora
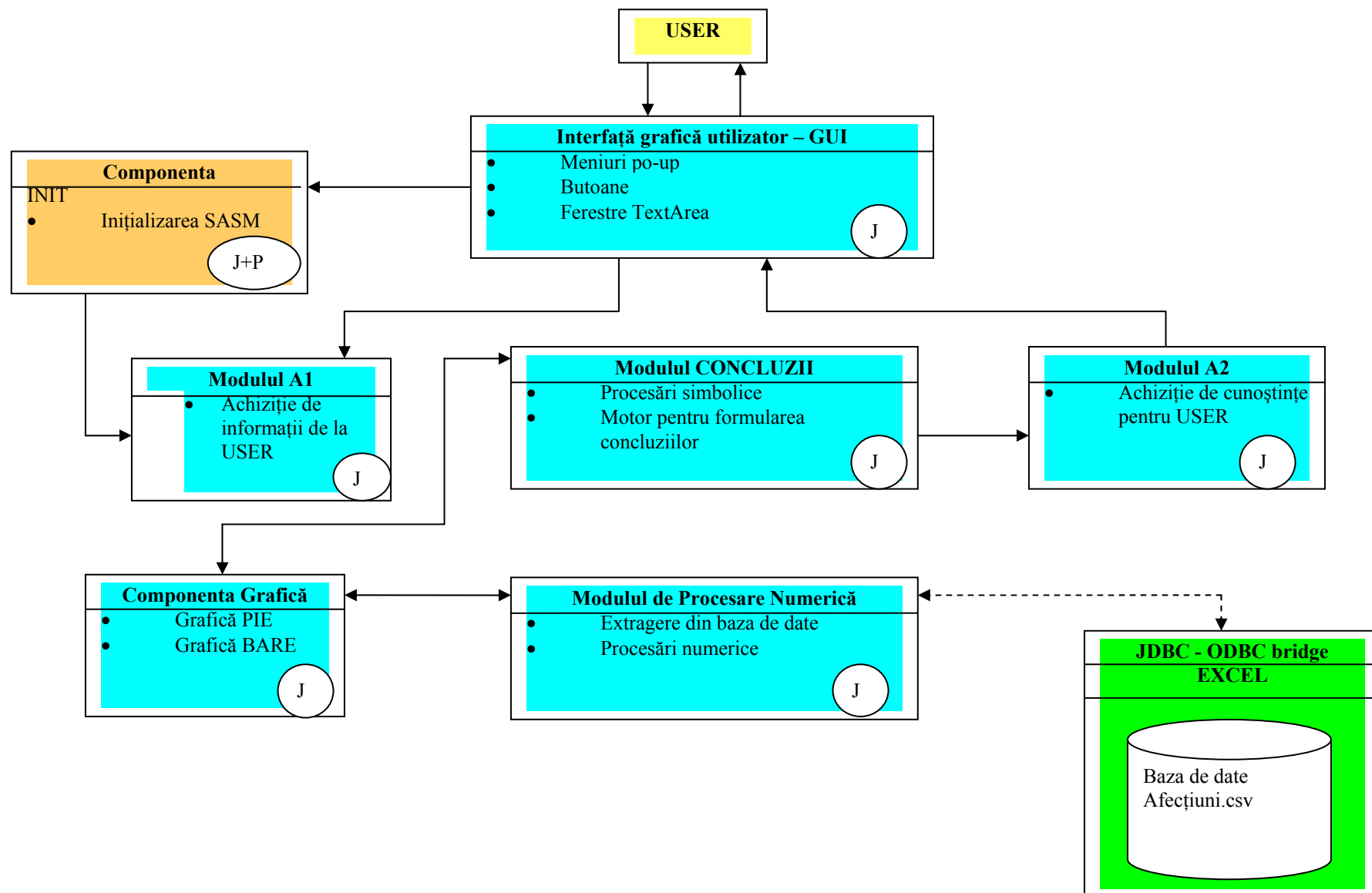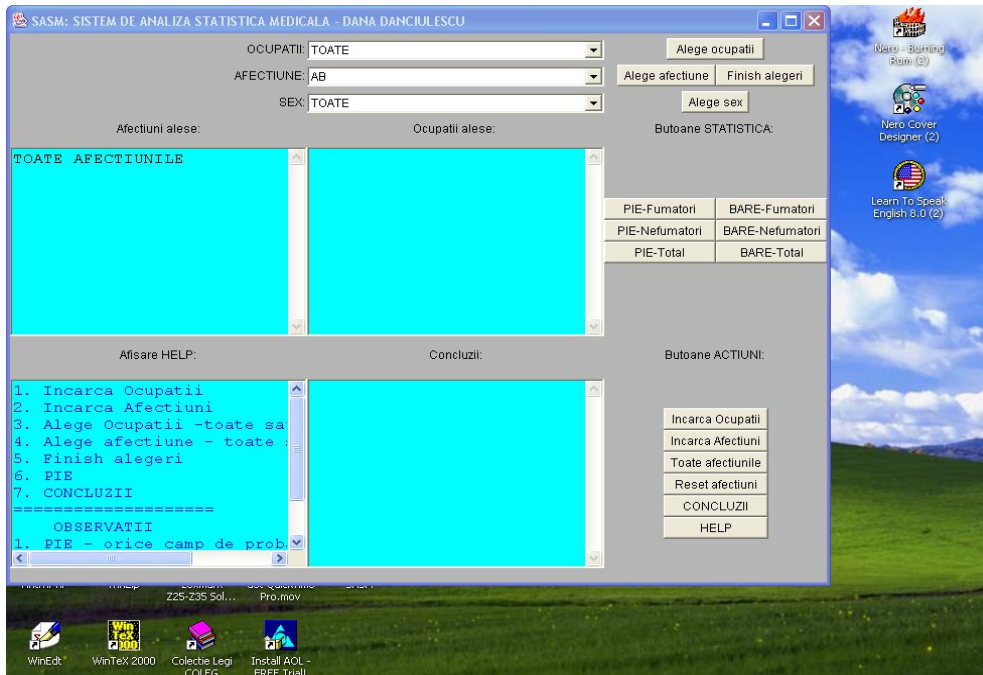
Picture 2
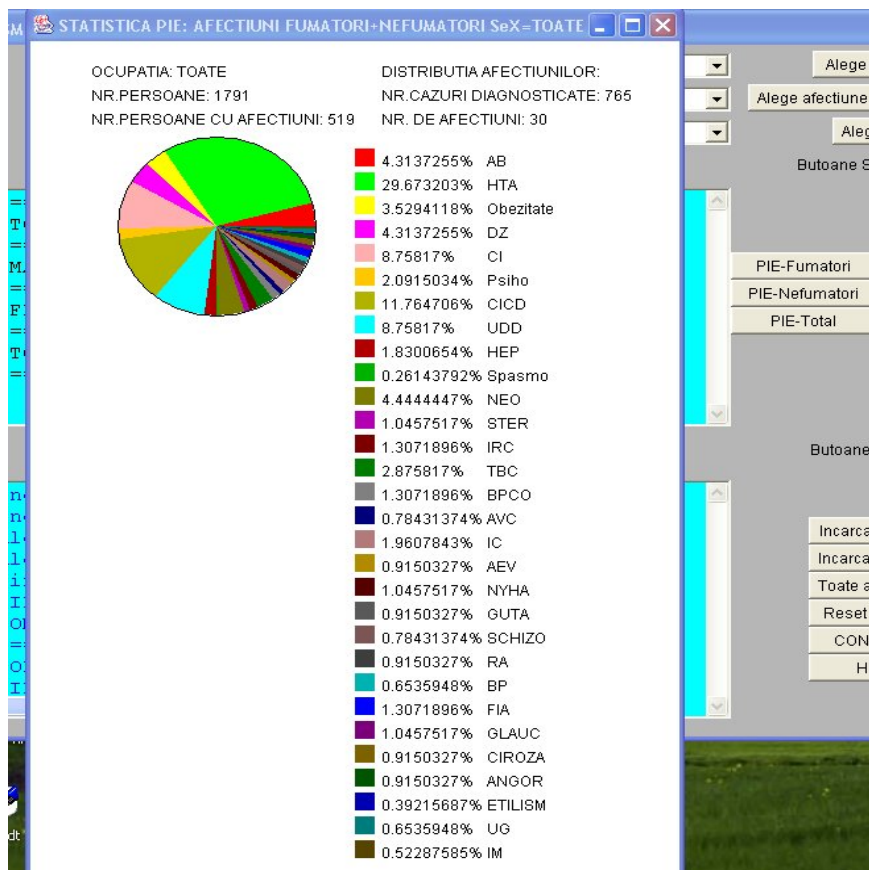
Picture 3
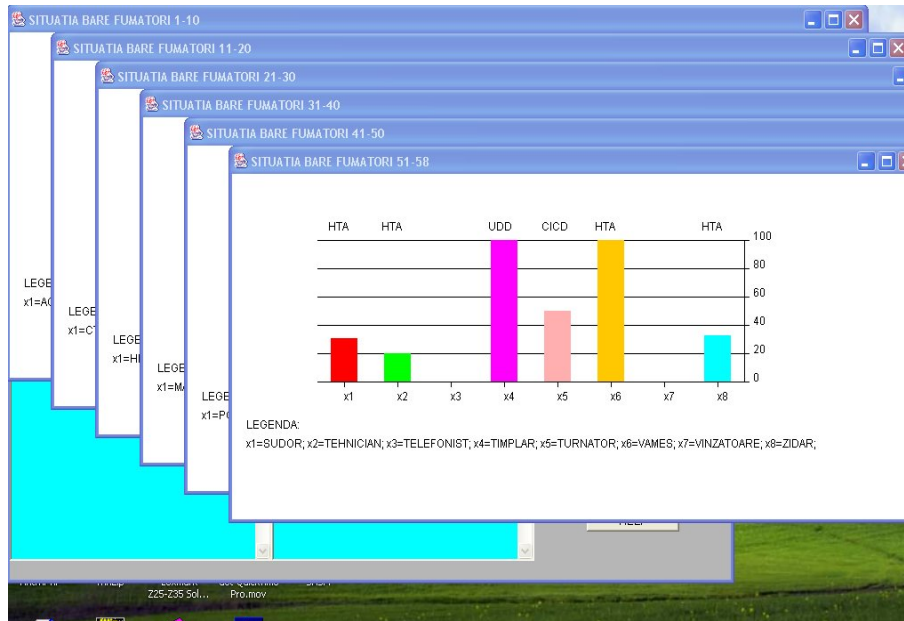
Picture 4

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INDUSTRIAL DATA ACQUISITION CENTRALIZATION AND DISPLAY

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Abstract: *Acquisitioned data from industrials process must be centralized and displayed in suggestive way. Only the most important data are displayed continuously. Data less important are displayed only at request. Waning messages are displayed in case of system malfunct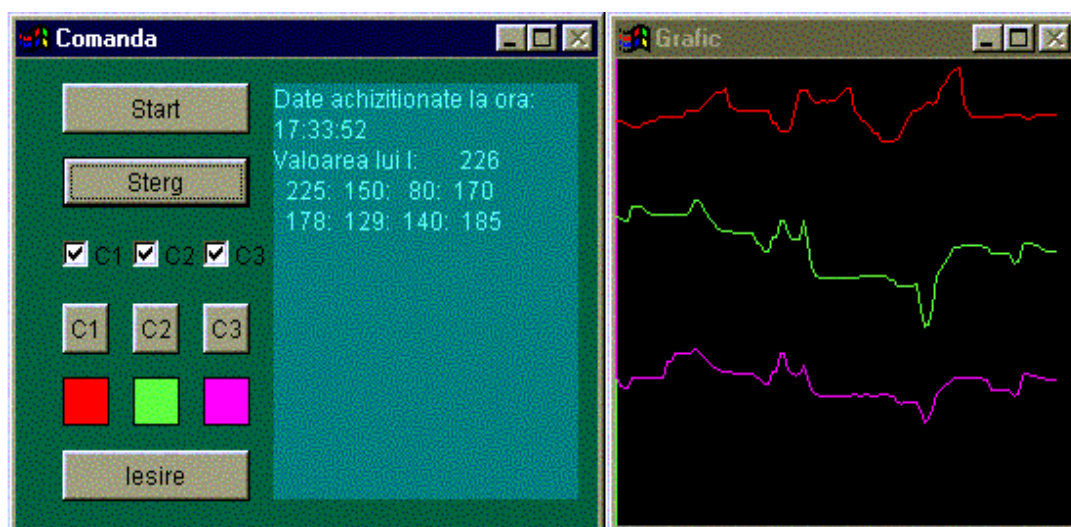ion. In case of many waning messages an intelligent system must decide which the important warning message is and must be displayed.*

A lot of industrials process needs automation, data transfer and data monitoring. Process control and monitoring are increasingly taking a higher profile in entire industry.

The complexity of industrials systems needs more sophisticated systems for control and data acquisition. The cabled systems can't keep up with tremendous evolution of industrial systems. The micro programmed system can adapt fast and continuous with evolution of industrial process.
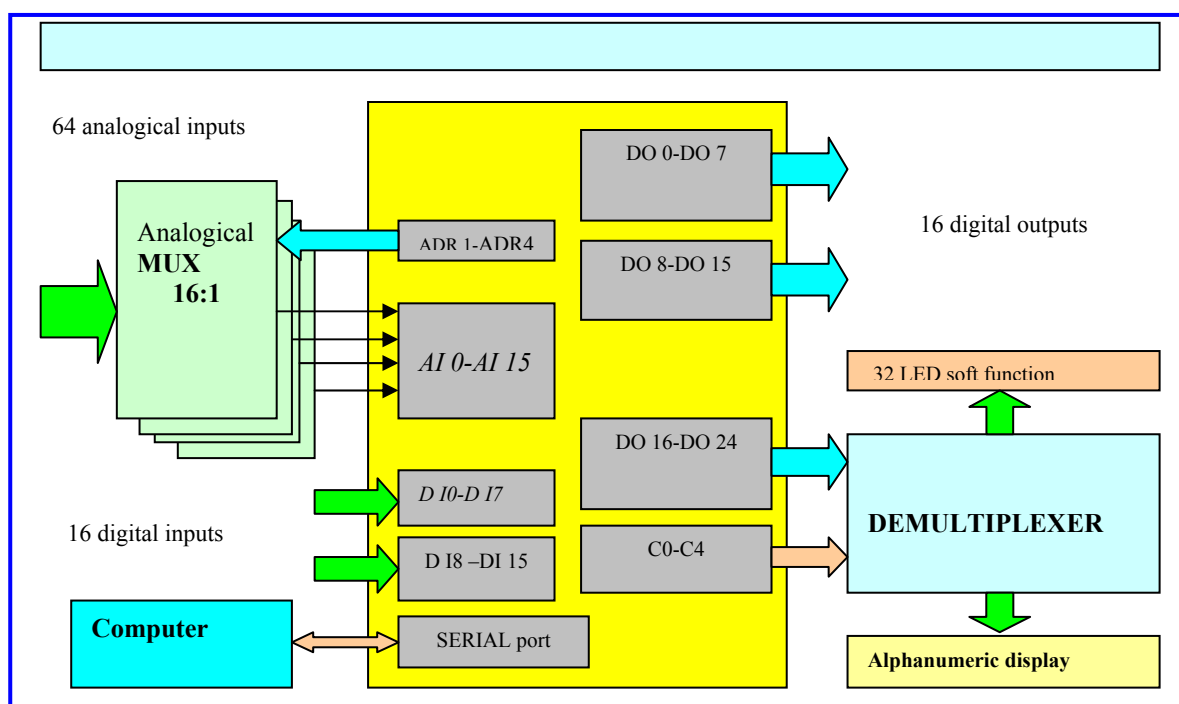
The micro programmed systems like microcontrollers can make data acquisition, leads industrial process according with the algorithm written in micro program, can signals different events using flesh light, can displays acquisitioned data using local display, can sends data to computer and can receives data and orders from remote computer.

Many industrial processes are controlled and monitored in decentralized mode. Every sub process is independently controlled and can communicate with a central point using serial data lines. Every sub system work independently and send data to central point only at master request

In case we want to transfer a large quantity of data at high speed, serial data transfer is inadequate. A better solution is using data acquisition through parallel port.

The parallel port has 8 bits data bus and a 9 bits commands bus is possible to make a faster data transfer.

This kind of data acquisition system connected through parallel port is shown in fig 2.
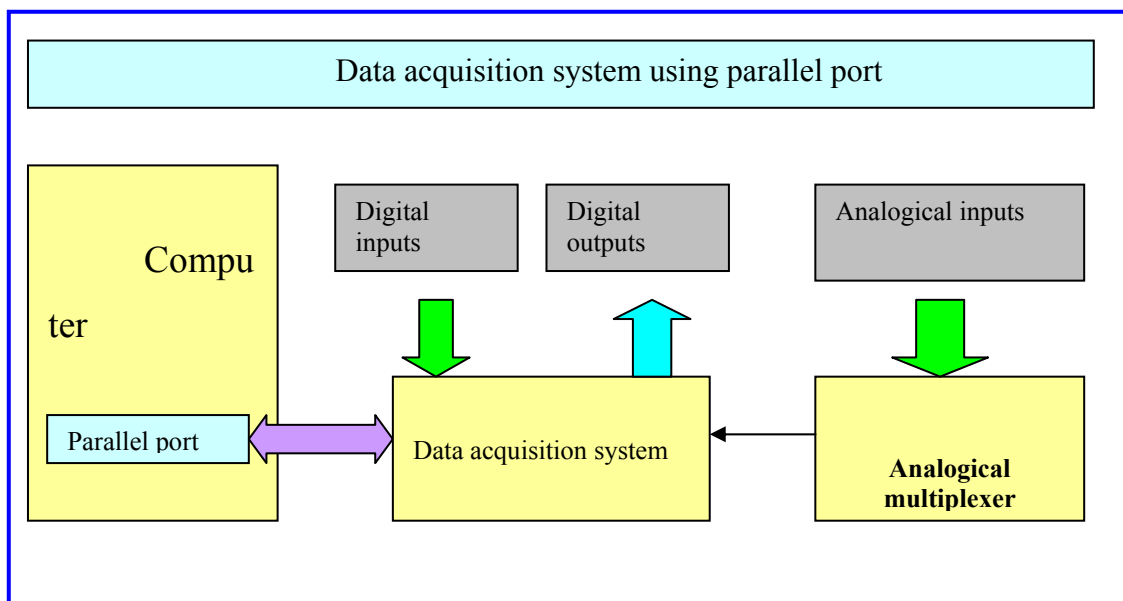
Figure 2

The data acquisition system can be an intelligent micro programmed system that works independently of computer or can be a simple interface using computer to make acquisition and data transfer.

If data acquisition system is a simple interface controlled by computer, the acquisition programs are only in computer, acquisition system is only an execution system. This kind of system is very simple, is cheaper and flexible. The computer can make complex local data displays.

For the industrial systems the speed of parallel data acquisition system is acceptable. The major advantage is lower price and simplicity of connection to the computer. This system is proper for control and monitors a large range of industrial process.

By example the screen of local debit measurement application is shown in figure 3.
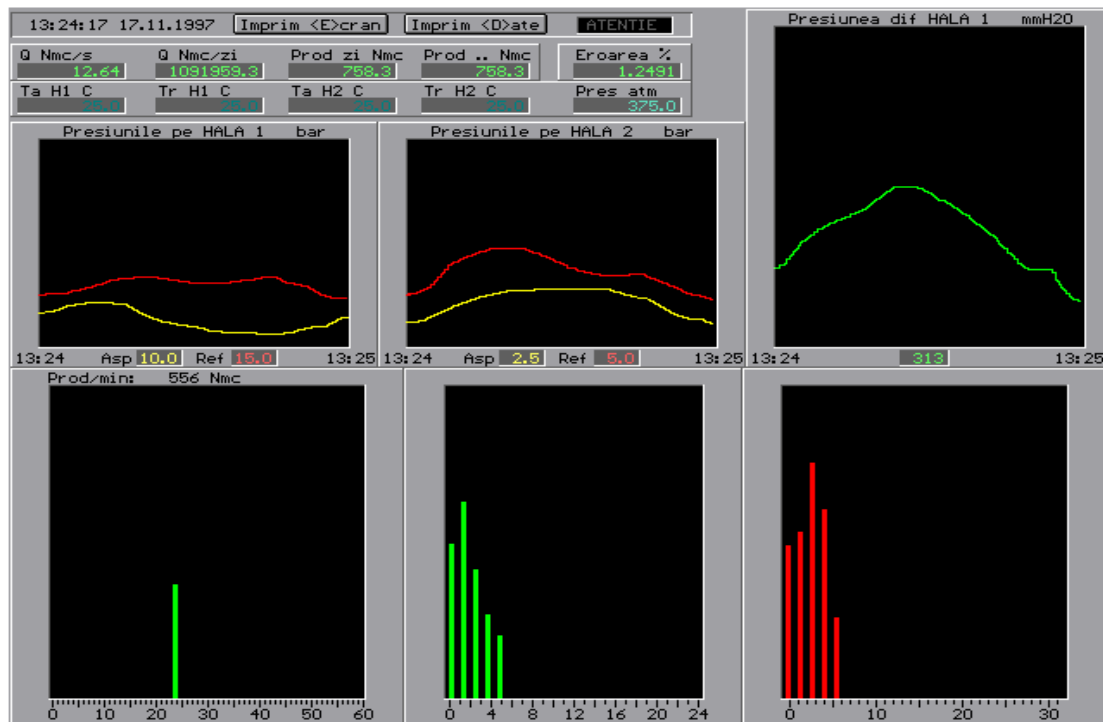
Figure 3

Other example of local data display and command is shown in figure 4

Figure 4

Acquisitioned data from industrial process must be centralize and display in suggestive way.

Only the most important data are displayed continuously. Data less important are displayed only at request. Waning messages are displayed in case of system malfunction. In case of many waning messages an intelligent system must decide which the important warning message is and must be displayed.

The program monitoring screen for 12 subsystems are sown bellow in figure 4.
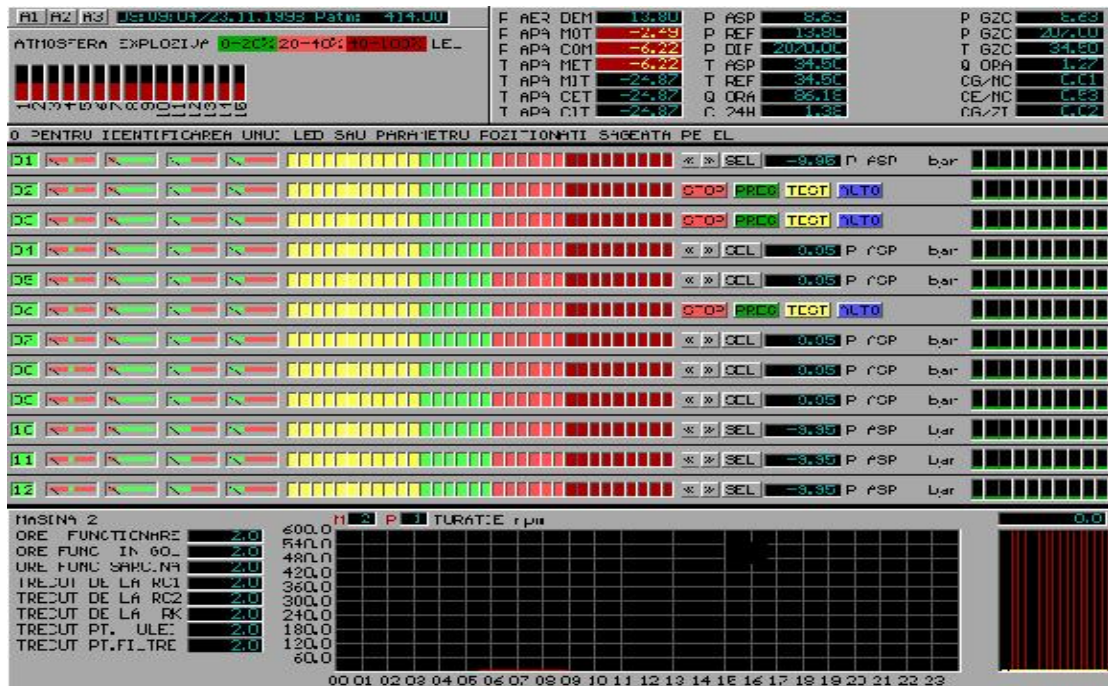
Figure 4

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A COMPONENT BASED APPROACH FOR SCIENTIFIC VISUALIZATION OF EXPERIMENTAL DATA

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Abstract: *The paper addresses the issue of component-based scientific visualization systems as a solution to most of the standard/commercial visualization system problems. The visualization process is requested to meet the process validation criteria. The visualization system is requested to observe a reference model. The benefits are discussed on a detailed component based visualization system.*

Index terms- scientific visualization, software components, visualization reference model, and visualization process validation

1. Introduction

The commercial visualization systems, the so-called "turn key systems", are easy to use but they have some disadvantages [1]:

- they are expensive;

- they run on pretentious platforms;

- they are "rigid", meaning that one can not use more than one instance of a visualization module. Several instances of the same module allow the simultaneous execution of several sets of input data;

- their modules can not be used to build other systems;

- the implemented algorithms are not always the algorithms desired by the user;

- the functionalities of a turn key system are "fixed", they can not be changed by the user.

This article proposes a new approach of the visualization field. Three goals are to be reached:

a. Description of a **reference model** for the visualization process;

b. Formulation of some criteria for the **validation of the scientific visualization process**;

c. Description of a **software component** model used to build personalized visualization systems. Such a model allows the rapid construction of visualization systems using modules that implement the desired algorithms.

The **reference model** represents an abstract view of the visualization process. It uses the concept of **level.** A level is seen as a distinct stage of the process that accepts data and services of a certain format as input. The result is a new set of data and services offered as input to the next level.

The reference model also serves to standardize the terminology, as well as to compare the visualization systems and to identify the constraints imposed by the process.

The **validation of the scientific visualization process** determines whether the process results in a *scientific* visualization.

The model of the **component-based visualization systems** is supposed to suggest some formalization capable to offer proper answers to the following questions: How does a software component behave? How are the interfaces between two components described? Which are the conditions that allow the composition of two or more components?

The idea of the component-based visualization systems is to use independent components to construct personalized visualization systems in order to obtain optimum and flexible systems that include a large variety of functionalities. A very important advantage introduced by this model is that the visualization systems can include user-made components, implementing the desired algorithms. The construction of components implies a large agreement upon the different data models, upon a formal model for time – sometimes the time being a critical variable -, upon a user model, etc.
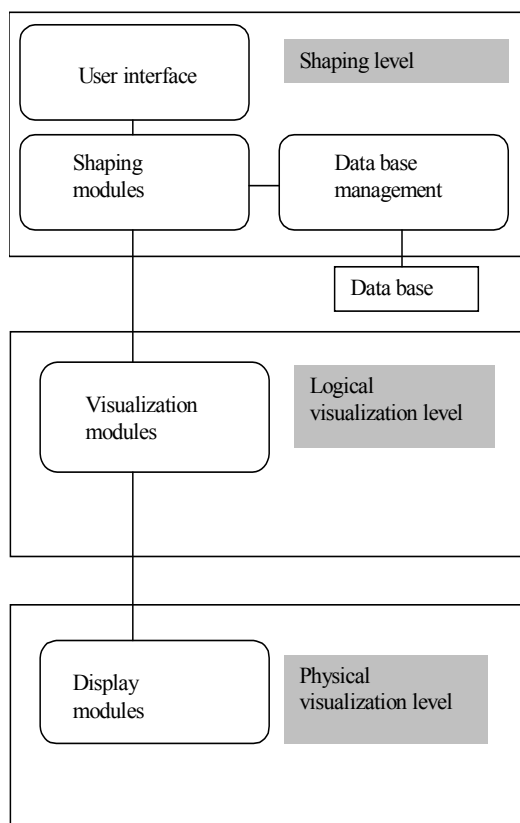
2. Towards a reference model

Fig. 1 The architecture of a visualization system for experimental data.

In order to describe the reference model for the visualization systems a "decomposition" of the visualization process is performed. The identification of the "levels" used for the data processing allows a better understanding of the conditions that must be imposed on the visualization function. The proposed model [2] contains three levels: ***modeling level, logical visualization level*** and ***physical visualization level.***

The ***modeling level*** only "extracts" from the data set the information of interest, allowing a global processing of the data. This level performs operations such as extraction of the data geometry, sub-sampling or re-sampling, extraction of the data set characteristics, etc.

The data objects are passed to the next level, the ***logical visualization level,*** where they are "factorized" into primitive data and mapped to glyphs and graphic primitives. A set of functionalities is implemented at this level, e.g. the choice of the visualization primitives, of the optical characteristics of objects, the computation of the optical properties of scenes, the light settings etc. Interaction elements are also present, performing rotations, translations, zooming.

The ***physical visualization level*** includes the choice of visualization medium and classical tools are available to perform the hidden surface removal, shading, lightning, etc.

Figure 1 proposes the architecture of a visualization system for experimental data following the above-described model.

3. Validation of the scientific visualization process

Scientific visualization is a computational process that transforms scientific data in visual objects [3]. Not all visualizations are scientific ones. A scientific visualization guaranties a certain degree of accuracy. In order to state conditions to be fulfilled by a scientific visualization, some mathematical structures over data sets are introduced.

3.1. Basic concepts and definitions

The idea of using the mathematical structures defined over data sets to find conditions imposed on the visualization function has been promoted by many authors [4], [5], [6], [7], [8].

Scientific data can be obtained in many different ways, e.g. by running a simulation or through a DAQ process. Usually, scientific data objects are finite representations of complex mathematical objects. We note by **O** the set of such objects, $o \in$ **O**. During the visualization process, initial data objects, o, are processed through different transformation functions *Mat(o) = o',* into a new set $o' \in$ **O'**. Objects o' are then mapped *Map(o') = g* into a set of ideal geometrical objects $g \in$ **G,** through a set of graphical primitives. Objects g are usually n-dimensional (nD), animated (t) and interactive. A group of g objects is usually called the **logical visualization** of a scene.

Ideal geometrical objects g, nD, animated (t) and interactive are usually represented *Rep(g) = g'*, $g' \in$ **G'**, on real 2D screens. A group of g' objects is usually called a **physical visualization** of a scene. Functions *Rep(g) = g'* implement classical graphical operations such as composition of the scene, volume generation, isosurface generation, simulation of transparency, reflectivity and lighting conditions, nD $\rightarrow$ 2D projection, clipping, hidden surface removal, shading, animation (t), setting user interactivity (zoom, rotate, translate, pan, etc), etc.

By **interactivity** we understand the attributes of visual objects (logical and/or physical) whose setting allows nD $\rightarrow$ 2D projection (zoom, rotate, translate, pan, etc), animation control (t), control of the objects composing the scene and control of the scene as a composite object.

The scientific visualization process is described by the *Vis(o)= g',*

Vis(o) = Rep(Map(Mat(o))) = g' function.

The above concepts and notations are synthesized in the table 1

Table 1

VISUALIZATION
Vis(o) = Rep(Map(Mat(o))) = g'
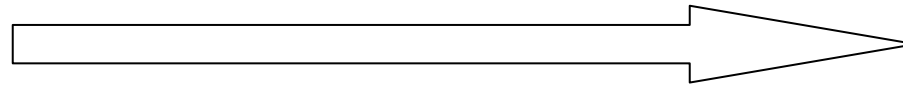
SCIENTIFIC MODELS	SIMULATION/ DAQ	SCIENTIFIC DATA	MODELING LEVEL *Mat(o)*	MODELLED DATA	LOGICAL VIS. LEVEL	VIRTUAL OBJECTS	PHYSICAL VIS. LEVEL *Rep(g)*	VIZUAL OBJECTS	INTERAC— TION *I(g')*
		O		*O'*	**Map(o')**	*G*		*G'*	
		o ∈ O	*Mat*(o) = o'	o' ∈ O'	*Map*(o') = g	g ∈ G	*Rep*(g) = g'	g' ∈ G'	
Model generation: Mathematical models based on ideal mathematical objects. Models are checked and improved by simulation and/or experimental measurements.	**Data acquisition:** (based on models) Aggregated scientific data objects.	**Defining models** for data objects "o" and **structures** over data sets, "O". Data objects are finite representations of complex mathematical objects.	Rigorous visualization (mathematical) **algorithms** applied to data objects filter initial values and/or generate new values (e.g. isosurface generation, volume generation, associating a color code to a scalar value)	Models and structures defined for input data should apply to **processed data objects**.	Processed data objects are mapped into ideal geometrical objects. The process is done through factorization of the complex data objects into primitive data and mapping them to gliphs and graphics primitives.	**Virtual geometrical objects.** A collection is such objects is called **logical visualization** of a scene. These objects are usually (nD), animated (t) and interactive (I).	The **rendering** process is done through traditional rendering algorithms: nD to 2D projection, rotation, translation, zoom, hidden surface removal, composition, animation, lightning.	**2D visual objects**, finite representations of the ideal geometrical objects. A collection is such objects is called **physical visualization** of a scene. These objects are usually (2D), animated (t) and interactive (I).	**Visual observation** of the scene and **interaction** (I), i.e. control of the visual objects and navigation. Human perception could be described in terms of mathematical strcutures.

3.2. Fundamental conditions of scientific visualization

There are many requirements concerning a certain process of scientific visualization. Here are the three fundamental ones.

The first one is the **distinctiveness condition**. This condition (although very weak) enables users to distinguish between different data objects based on their display. The condition is necessary as one can imagine many visualization functions that generate images with no use, which reveal none of the data objects characteristics/attributes.

The second condition is the so called the **expressiveness condition.** It assures that the attributes of the visual object represent the attributes of the input data set.

The third one is the **precision condition**. This condition insures that the order among data objects is preserved among visual objects.

The distinctiveness condition

Different input data (different mathematical objects) have to be mapped into different visual objects.

This can be stated as:

$o_1 \neq o_2 \Leftrightarrow Vis(o_1) \neq Vis(o_2) \Leftrightarrow Rep(Map(Mat(o_1))) \neq Rep(Map(Mat(o_2))) \Leftrightarrow g_1' \neq g_2'$, for any $o_1, o_2 \in O$, $g_1', g_2' \in G'$

The interpretation of this condition is that *Vis(), Mat(), Map()* and *Rep()* functions are injective.

The expressiveness condition

The visual objects express all and only the characteristics of input data.

It results that the visualization function should be bijective/one to one.

The two conditions are necessary but not sufficient. Another condition is needed to establish an order relation, seen as a precision relation.

The precision condition

For any objects $o_n, o_m \in O$ such that o_n is "more precise" than o_m we have $Vis(o_n)$ "more precise" than $Vis(o_m)$, with $Vis(o_n), Vis(o_m) \in G'$.

The precision relation adds something new. If the visualization function is well defined and the input data objects are strictly ordered, the visual objects can be ordered by "precision".

The first two conditions introduce criteria of validation and control of the visualization process. The visualization function *Vis()* fulfilling these criteria results in a scientific visualization.

The third condition allows further developments by defining mathematical operations on the given ordering.

The mathematical structures and the other notions presented above are later used to support the main ideas of this paper.

4. Component based visualization systems

The alternative to the acquisition of commercial visualization systems and their parameterization to fit to the user's problem is the use of components. These are pre-made complex functional entities, that can be (re)used to construct the desired architecture of a visualization system. A component performs a specific visualization task, which can be isolated inside the system.

A component can be a class or a collection of classes. Unlike classes, a component can be implemented using a technology that is completely different from the object oriented one, e.g. an assembling language. Classes are some how similar to components, but the object-oriented technology has not imposed or hasn't been able to impose components. One of the reasons is that the definition of the objects is a purely technical one. An object represents the encapsulation of a state, of a behavior, polymorphism and inheritance. The definition does not include the notions of independence and forward composition. Components as well as objects are re-usable. Components' description has to be more general in order to be capable to assure their independence and reusability.

Fig. 2 *The architecture of a component based visualization system*

Finally, the interaction between components has to lead to a functional system.

Considering the reference model introduced in the section 2, we conclude that a component-based visualization system resembles figure 2, where each rectangle represents a component.

4.1. Software Components

4.1.1. Basic Notions and Definitions

A *software component* is a binary program that represents the physical encapsulation of related services according to a published specification. [9]

A software component can be seen as an interactive system that communicates asynchronously through channels. The services implemented can be accessed through a consistent and published interface that includes an interaction standard. A component has a *black-box* view captured by the published specification, and a *white-box* view showing implementation details.

The interface defines **a set of channels C,** divided in **a subset** $A = \{A_1, A_2, ..., A_n\}$ **of the input channels** and **a subset** $B = \{B_1, B_2, ..., B_m\}$ **of the output channels,** meaning that **C=A ∪ B.** A component can be described graphically as in figure 3.
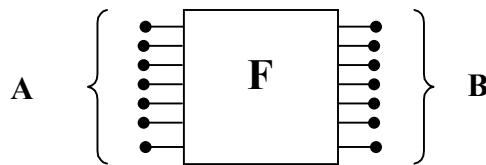
Fig. 3 *The graphical description of a software component*

Any component implements a function F:

$$F : X^n \rightarrow Y^m$$
$$\left(x^1, x^2, ..., x^n\right) \rightarrow \left(y^1, y^2, ..., y^m\right)$$

which assigns an output on *m* channels to an input on *n* channels. In other words it assigns the output data $(y^1, y^2, ..., y^m)$ to an input data $(x^1, x^2, ..., x^n)$. The inputs are timed streams of messages. The component sends new outputs every time the inputs change. This is why a software component can be seen as a system that communicates asynchronously (the component waits for a message; when it is received the component processes it and the result is sent to the output; then, the component waits for the next message). A message can be a string of characters, a binary number, a decimal number etc. Each channel has a stream of messages attached, representing all the messages received (sent) through that channel.

A component needs a time τ in order to process a message. We consider that the inputs are introduced at the given moments: $t_0=0$, $t_1=\tau$, $t_2=2\tau$,..., $t_n=n\tau$. That means that the n-th message is accepted by the component only at the t_n moment.

Consider the input stream of messages *x*. The following notations are introduced:

- *x(n)* - the string of the first *n* messages (until the moment t_n) from *x*;
- x_n^i - the input message on the channel *i* at the moment t_n;
- F(*x*) - the *stream* of messages assigned to the output channels for the input *x*;
- F(*x*)(*n*) - the first *n* messages from F(*x*);
- $F_n^i(x)$ - the output message on the channel *i* at the moment t_n;

- F_x - the *set* of messages assigned to the output channels for the input x;

- A_x - the subset of the input channels for which the input is different from zero, assigned to the input x ($A_x \subseteq A$);

- $B_{F(x)}$ - the subset of the output channels for which the output is different from zero, assigned to the output $F(x)$ ($B_{F(x)} \subseteq B$).
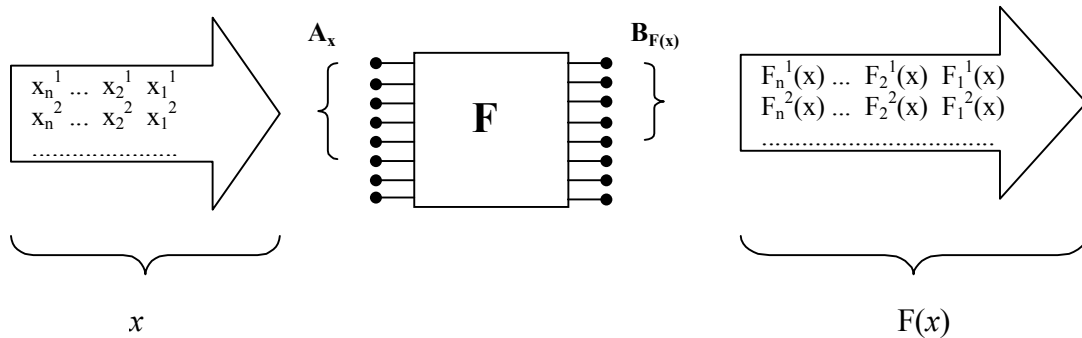
The way a component works is described below (Figures 4),
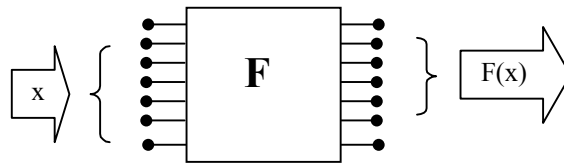

Fig. 4 *The way a software component works*

or simplified (Figure 5).

Fig. 5 *A simplified scheme of the way a software component works*

The following definitions are introduced:

*(a)The function F is **consistent** if for x = z $\Rightarrow$ F(x)=F(z), for any streams of messages x and z.*

The function F is consistent if for identical inputs, identical outputs are obtained.

(b)*The function F is **causal** if for any n natural the following condition is fulfilled:*

$x(n)=z(n) \Rightarrow F(x)(n)=F(z)(n).$[9]

If the function F is causal then it is also consistent.

The processing time for a causal function is τ=0. Such a component can not be constructed. We call it ideal.

*(c) The function F is **strictly causal** if for any n natural the following condition is fulfilled:*

$x(n)=z(n) \Rightarrow F(x)(n+1)=F(z)(n+1).[9]$

We consider F(x)(0)=F(z)(0).

A real component is always strictly causal, meaning that the output at the moment t_{n+1} corresponds to the input at the moment t_n. This component can be implemented because the processing time is $\tau>0$.

*(d) The function F is **realizable** if there is a function strictly $f : X^n \rightarrow Y^m$ such that for any input x we have $f_x \in F_x$ (the output determined by f belongs to the set of the outputs determined by F).[9]*

We denoted by f_x the set of messages assigned by the function f to the output channels, for the input x.

A function f with $f_x \in F_x$ for any input leads to a deterministic behavior of the function F (Figure
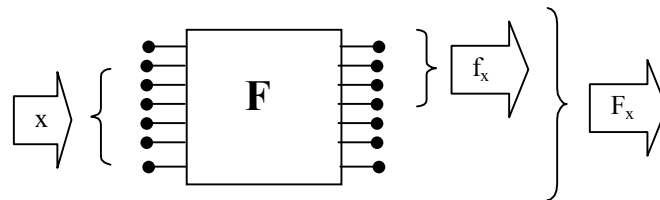
Fig. 6 *A function f with fx ∈ Fx for any input leads to a deterministic behavior of the function F*

*(e) The function F is **fully realizable** if it is realizable and for any input x there are **p** functions strictly causal $f_x^i : X^n \rightarrow Y^m$, i=1,...,p such that $F_x = \bigcup\limits_{i=1}^{p} f_x^i$.[9]*

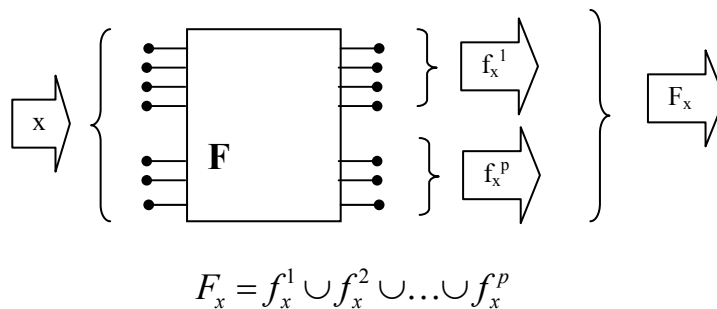$$F_x = f_x^1 \cup f_x^2 \cup ... \cup f_x^p$$

Fig. 7 *Graphical description of a fully realizable function F.*

This property guarantees that for every output there is a strategy (a deterministic behavior) that produces this output (Figure 7).

*(f) The function F is **time-independent** if the timing of the messages in the input streams does not influence the messages in the output streams but only their timing. [9]*

4.1.2 Composition Rules

Channels are assumed to have a type from a given set of types T assigned. A type is a name for a set of data elements. We consider the function *type* that assigns a corresponding type from the set T to each channel from the set C (type: C→T).

Definition: *Two components that implement the functions F_1 and F_2, are said to be **composed** if output channels of the first component are used as input for the second one* (See Figure 8).
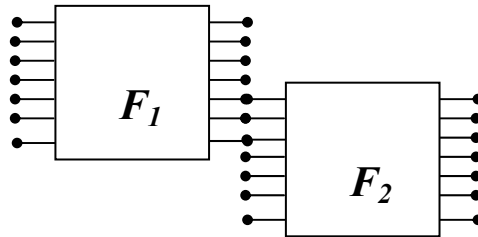

Fig. 8 *Two composed components*

Remarks:

▪ Finally the two components can be seen as one (Figure 9).
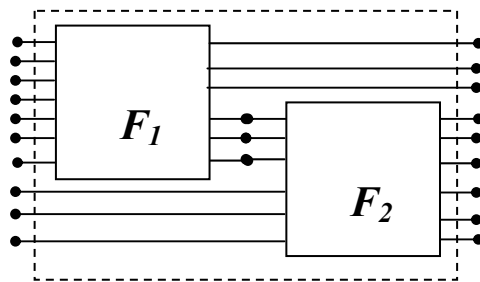
Fig. 9 *Two composed components seen as one.*

▪ The definition can be restated for more than two components (Figure 10).
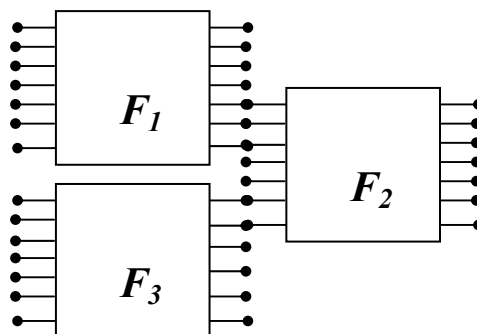
Fig. 10 *Three composed components*

We can say that the composition of two components is reduced to the existence of common elements in the sets B^1 and A^2, where B^1 represents the set of output channels of the first component A^2 represents the set of input channels of the second component. We can state that two components are composed if $B^1 \cap A^2 \neq \emptyset$.

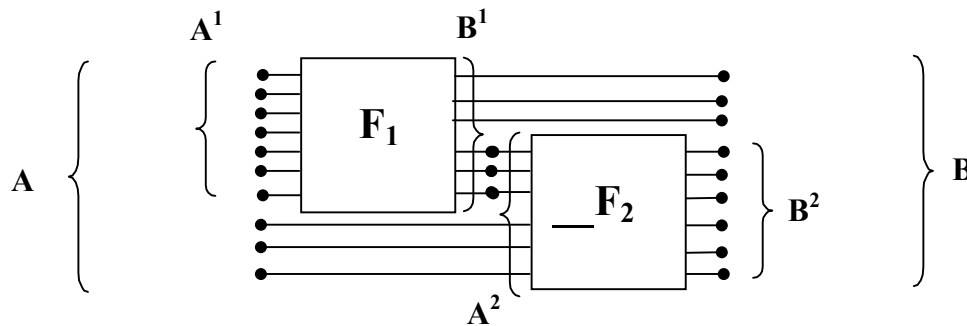**Condition.** Two channels, one output B_n^1 and one input A_m^2, can be used for the composition of the components if and only if *type (B_n^1)=type(A_m^2)*.

Fig. 11 *A detailed view of two composed components.*

Consider two composed components (Figure 11).

The following properties can be stated:

a) $A = A^1 \cup (A^2 - B^1)$, $B = B^2 \cup (B^1 - A^2)$, $C = A \cup B$.

b) Considering that the processing time for the first component is τ_1 and for the second one τ_2, the processing time of the composed component is, for the worst case, $\tau = \tau_1 + \tau_2$.

$$F = \begin{cases} F_1 & \text{,if } A_x \subseteq A^1 \text{ and } B_{F(x)} \subseteq (B - B^2); \\ F_2 & \text{,if } A_x \subseteq (A - A^1) \text{ and } B_{F(x)} \subseteq B^2; \\ F_1 \circ F_2 & \text{,if } A_x \subseteq A^1 \text{ and } B_{F(x)} \subseteq B^2. \end{cases}$$

c) Using the notations from the first part of the article we describe the function that implements the composed component as follows:

Remarks:

▪ $F_1 \circ F_2$ means that for an input *x* we get:

$$x \rightarrow F_1(x) \rightarrow F_2(F_1(x)) \equiv F(x)$$

▪ The function described above does not include all cases. For example, the function is not defined for $A_x = A$ and $B_{F(x)} = B$. These cases are solved by decomposition, in order to fit into the given description.

4.2. The Visualization Pipeline Using Components

Consider the case of one input -one output component. The function F describing the component is considered to be time-independent (*definition (f)*). Two working steps can be revealed: at moment t_0 a message is introduced into the input channel and at moment t_1 the result is read from the output channel (Figure 12)
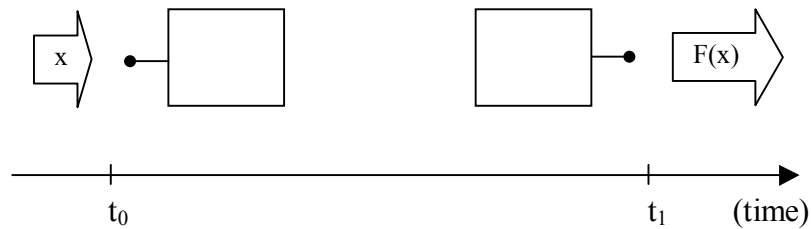

Fig. 12 *A one input-one output component.*

We considered that the implemented services are **consistent** (*definition (a)*). The component is **strictly causal** (*definition (c)*) and **fully realizable** (*definition (e)*) because the strictly causal function needed is F.

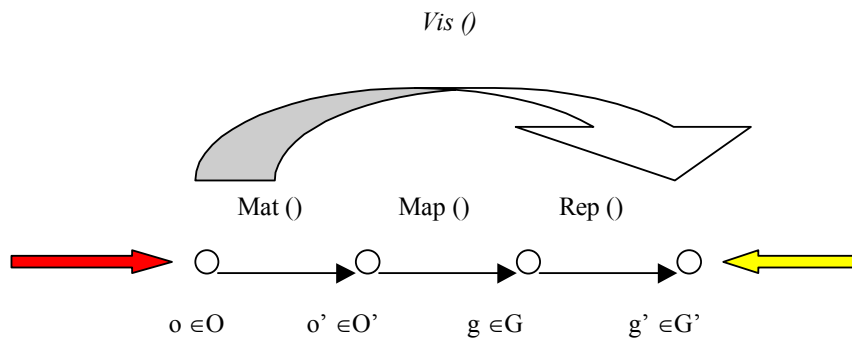The reference model introduced to describe the visualization process contains three distinct steps (Figure 13).

Fig. 13 *Synthesis of the proposed visualization model.*

Suppose that each step/function is performed by an independent component. Then the visualization pipeline becomes the one in figure 14. The components should take into account the scientific visualization criteria, meaning that the *Mat(),Map(),Rep()* functions implemented by the components should be injective and *Vis()* should be bijective.
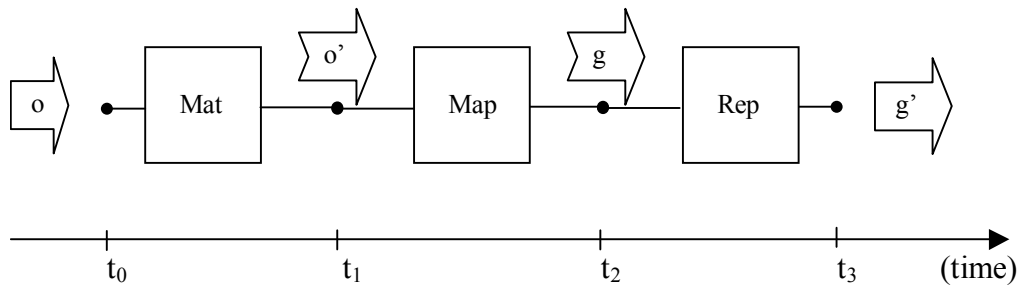
Fig. 14 *A component-based visualization system.*

A further decomposition can be performed. Each component can be obtained from the composition of two or more components depending on the visualization process. A personalized visualization system is obtained at this level.

4.3. Example: Visualization of natural gas reservoir using a component based visualization system

In order to present a detailed component based visualization system we consider the example of a natural gas reservoir. A nonuniform data set obtained using drills is visualized. The
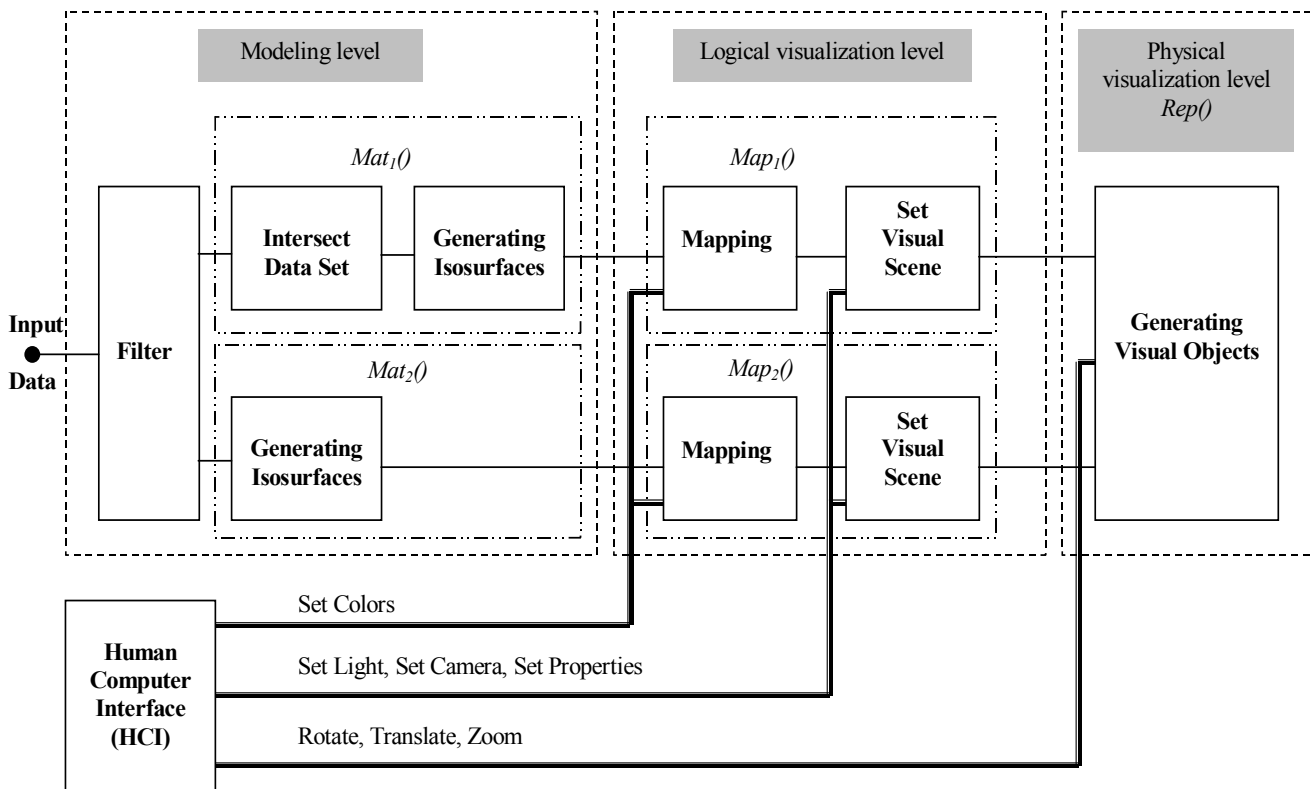

Fig. 15 *The architecture of a custom made system used to visualize a natural gas reservoir*

data set is processed using two "paths"(Figure 15).

The "filter" component transforms the nonuniform input data into a uniform data set using the geological characteristics of the underground and the methane concentration. The filter also performs the sub-sampling of the data set. One of the "paths" includes the intersection of the data set with a plane. The generation of the isosurfaces represents another step of the modeling level.

The colors, the light, the position of the camera and other properties are set at the logical visualization level. A human-computer interface (HCI) allows the user to change the properties without interfering with the data flow.

Finally, two objects that can be rotated, translated and zoomed are obtained (Figure 16): one used for the dimensional exploration of the deposit and another for the visualization of the level lines.

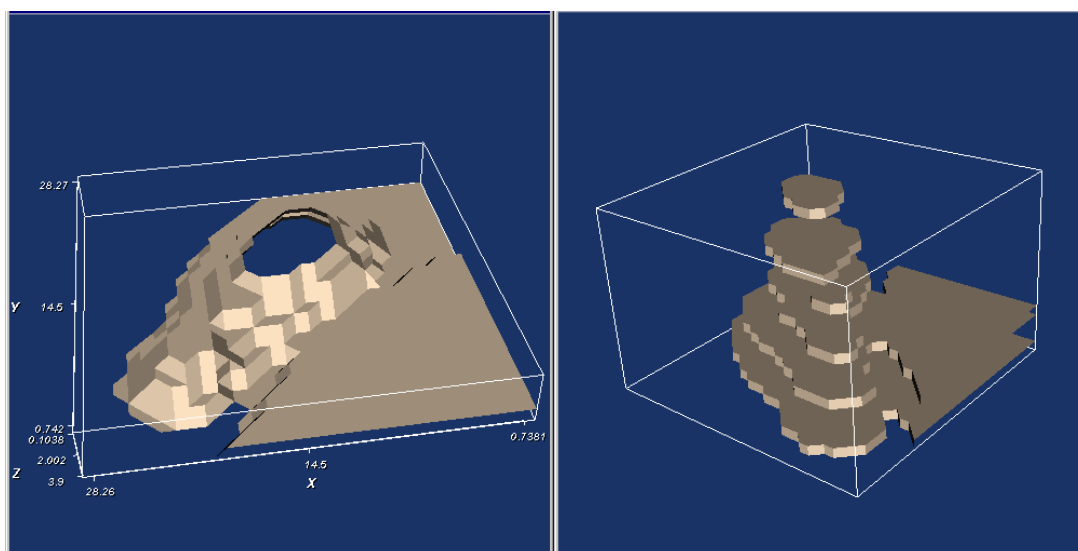(The presented images were obtained using the VTK library, and the components were implemented using Tcl/Tk)

Fig.16 *The visualization of the natural gas reservoir*

5. Conclusion

The paper addresses the issue of component-based scientific visualization systems. The visualization process validation criteria are presented in detail and a reference model for the visualization systems is proposed. The paper supports the idea that a component-based architecture solves most of the standard/commercial visualization system problems. It should be noted that the introduction of the component-based technology does not solve lead automatically to the wide spread of customizable scientific visualization systems unless a critical mass is reached.

The theory is supported by a detailed example of a visualization application of a natural gas reservoir.

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SECURE E-BANKING

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Abstract: *The interaction between user (home user or corporate) and bank (domestic or international) has been substantially improved by deploying e-banking applications. This paper discusses the security of today's electronic banking systems in Romania.*

Key words: *e-banking, XML, WML*

1. Introduction

Technology details

E-banking systems give everybody the opportunity to access every day: 24 to 24 hours, 7 to 7 days and 365 days per year to their banking activities retrieving an account balance, money transfers (between a user's accounts, from a user's account to someone else's account), retrieving an account history.

To enable mobile banking and mobile commerce implementations, both server side and device side components are required. The primary challenge on the technology side is to ensure that Internet-transmitted data and display information is optimized/converted for wireless networks through WAP gateways and is reformatted for final display on consumers' wireless handsets.

On one hand, the server side components are required by application providers, wireless networks and hosting financial institutions. The server hosts a corporate application gateway and something called the reformatter to make it possible to convert screen information for display on mobile devices. Network gateways are also essential for connecting TCP/IP corporate networks to wireless networks that provide greater bandwidth.

On the device side, mobile users must have handsets that support mobile browsers called micro browsers to support other handheld wireless computing devices. Through the network gateways mentioned above, users provide a connection to the transport layer that pushes relevant data and screen display information. Interpreters ensure that the information delivered in standard formats such as HTML, WML, XML, etc. can be converted for presentation on a micro-browser or other display interface.

Lastly, on the consumer side, the most important part of the equation is the mobile micro browser that is embedded in wireless-enabled mobile phones. The biggest challenge financial institutions will face is the ownership of embedded portals that wireless subscribers will view each time they use their mobile.

What about the security issue in a mobile environment?

As mentioned earlier, security continues to be a big barrier for Internet transactions so it will be a bigger concern in the wireless environment. To ensure success as a device fully capable of transacting, security standards will need to play a leading role in swaying consumer confidence and mitigating the fear of fraud. PKIs seem to be the favourite solution for security as it is considered the best and by and large impossible to break.

PKI (Public Key Infrastructure) technology provides a security solution that is superior to standard SSL, in that it protects both the integrity of the data stream in addition to verifying the participants engaging in a private, confidential transaction. In simple terms, PKI involves private and public keys shared between two parties engaging in a secure transaction. A third element, the digital certificate, is signed by a third-party certificate authority.

Only when the technology back-end employed and the security measures implemented are foolproof and truly convincing, will a consumer venture out into newer horizons. And mobile banking is one such service that has tremendous potential and will witness colossal growth in the years to come.

2. Scope

This paper discusses the security of today's on-line banking systems; It concentrates on Internet My work is based on an extensive survey of the available on-line banking systems in Romania; I've investigated the security of the electronic banking systems of around 10 banks.

There are two main players on the Romanian e-banking market: Soft Net and Net Consulting.

The internet e-banking structure is possible to view in the follow diagram
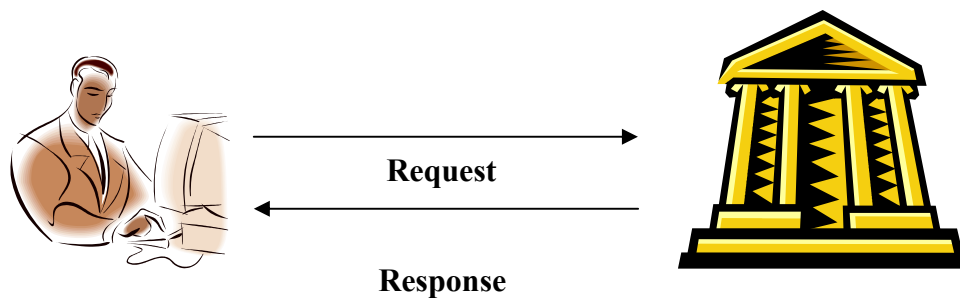
Picture 1 e-banking view

The technology is "state of the art" IBM Lotus Notes or Oracle Platform, Java like programming language; security: https, EDIFACT and SSL corroborate with SWIFT for non domestic transactions but the connection is Dial-Up. Companies are used Dial-Up connection for the e-banking application from two points of view. The IT administrators for the companies who are using e-banking application do not agree an Dial-Up connection from the internal LAN or Intranet and on the other hand an Dial-Up connection has three payers: first the user, second the communication company (Romtelcom, ZAAP, Connex, Dialog) an third the bank. When we have three tier or multi tier architecture, the security risks grow.

Bank	Supplier	Technology	Security	Connection
BancPost, EximBank	Soft Net	Lotus/Notes IBM Java	VASCO Digipass [1] https, SSL e-signature IBM SecureWay	Dial-up
ABN AMRO Bank Banca Comerciala Carpatica BCR B.R.D. Groupe Société Générale CitiBank ING Barrings Bank	Net Consulting	Oracle Java	SWIFT [2] EDIFACT [3] FTP e-signature	Dial-up or www

3. Concluding remarks

As technology evolves, more of our daily life's activities are moved on-line. Electronic banking is an important example of this trend. The issues discussed in this paper are generally applicable to other e-services.

A first main security issue consists of the establishment of a secure channel to provide data confidentiality and data integrity of communications between a client and an authenticated bank. The second security issue is the authentication of the client, at the beginning of a session, i.e., entity authentication, and for each transaction, i.e., transaction authentication. With respect to the first issue, almost all today's electronic banking systems rely on the https/SSL protocols. Although similar solutions would be possible, this is certainly the most popular alternative, as it is available in all common web browsers. From a cryptographic point of view, SSL has proven to be a practically secure protocol. With respect to the second issue, fixed passwords are still most widely used, as they are easy to implement and use. From a security point of view, public-key cryptography is the best solution, and the only one providing non-repudiation.

Even the best solution relies on the assumption that the end-points of the system are trusted. Concerning the client, this means that the electronic banking system should run within a trusted environment which has not been tampered with. This is however far from trivial in a typical client environment. The user must be responsible for maintaining his computing environment, and keeping it trustworthy. The user must also protect the secret elements that are used in the system, such as passwords, private keys, or tokens. While inherently more secure environments will considerably improve the overall security of an electronic banking system, education of users has an even more significant impact on its overall security. Banks on their turn should ensure the security of their servers.

Security is all about risks and associated costs. In an electronic banking system, the cost of security measures at the client side is reduced as much as possible, while keeping a minimal level of protection. One cannot achieve perfect security, so at some moment in time, despite all security measures something will always go wrong. The real question is what will happen in that case Banks of course will want to pass liability to their users and vice versa. Law and small print on contracts will somehow make clear who will have to take the risk in the end.

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MULTIPLEXED SYSTEMS FOR SERIAL DATA LINES

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Abstract: *Many industrial processes are controlled and monitored in decentralized mode. Every sub process is independently controlled and can communicate with a central point using serial data lines. Multiplexed systems for serial data lines are used to connect many serial data lines to central point.*

In decentralized processes control every sub process is permanently controlled by his own automation system, that make local control and parameters display and must communicate with central point in order to transfer acquisitioned data and receive commands. For a large number on sub industrial systems connected to central point it must use a simply and efficient way to communicate with central point.

The most efficient way to connect all sub systems is a single serial line connection is multi dropping system (figure 1).
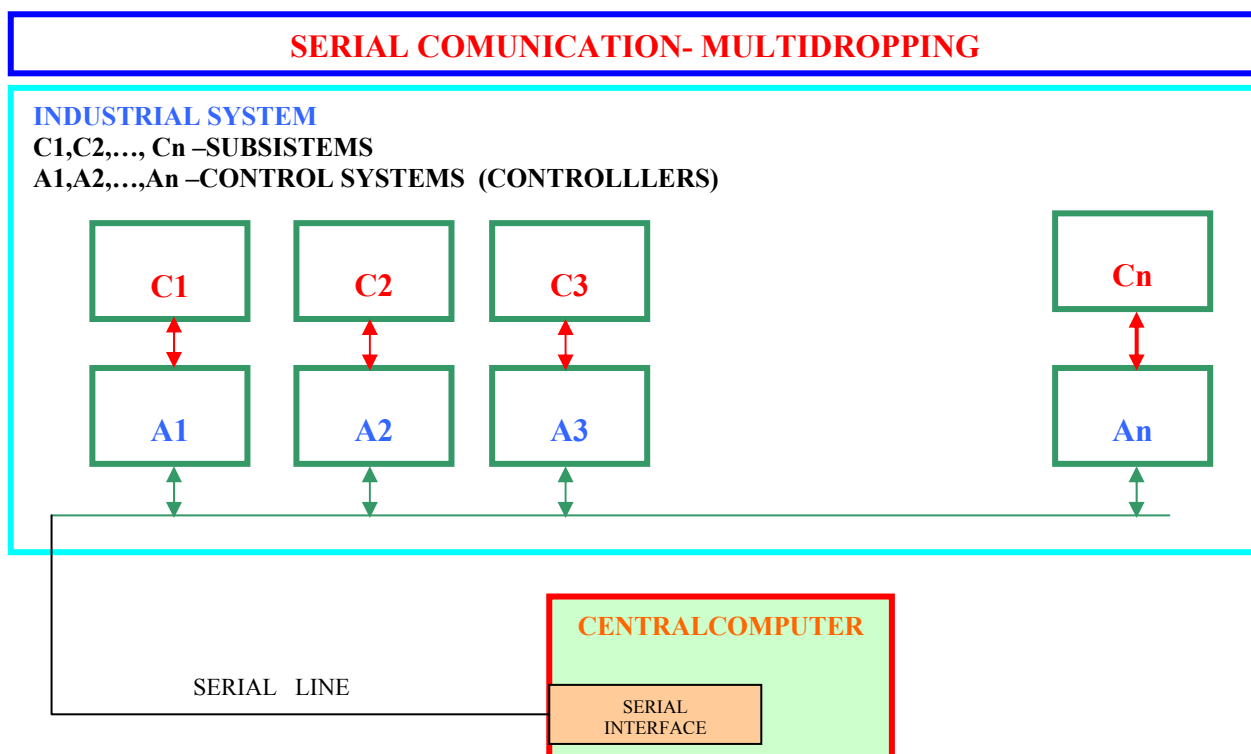
All sub system connection trough a single serial line is possible only if every sub system has multi dropping facilities.
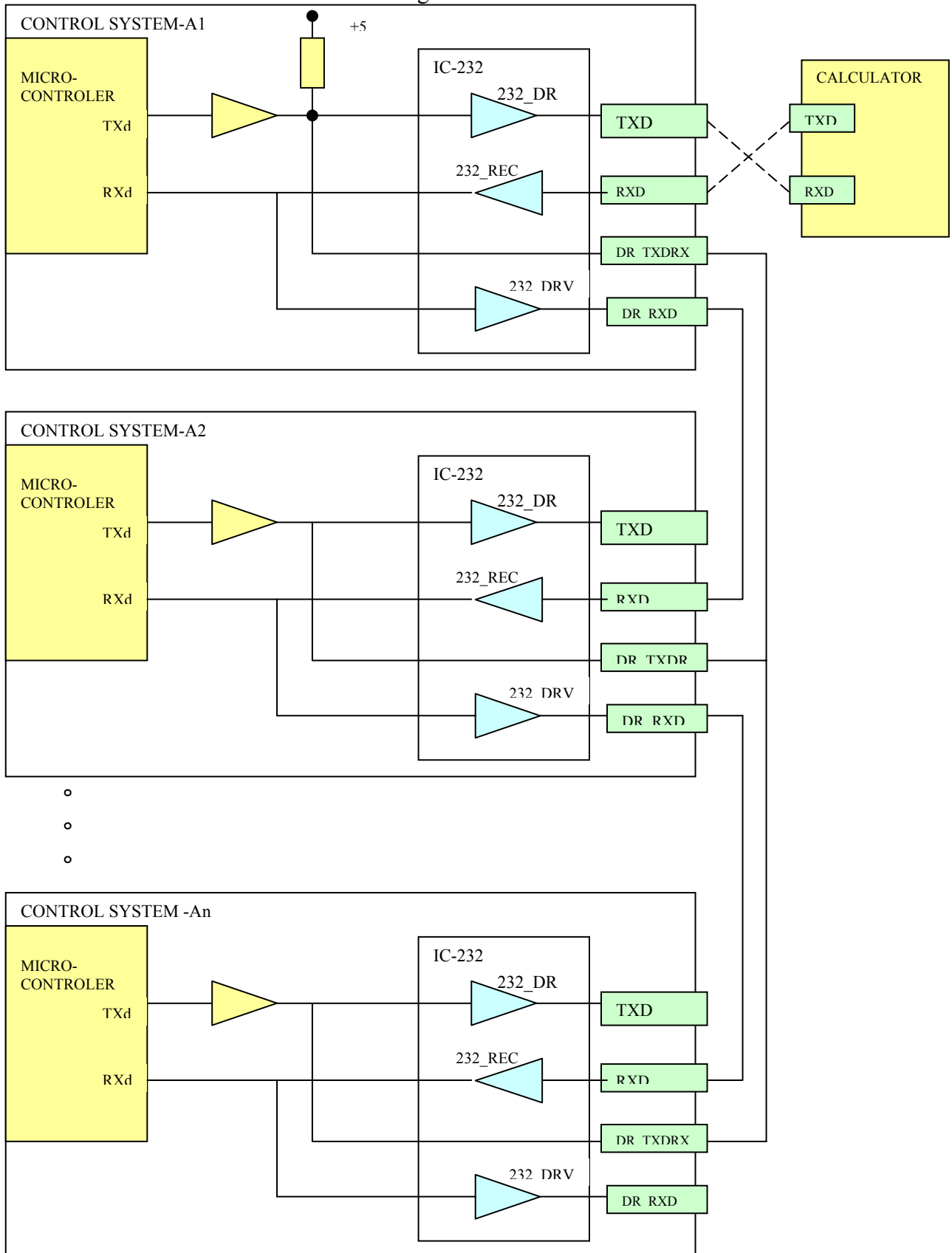
These facilities are shown bellow in figure 2.

Figure 2

The complexities of lies between sub industrial systems and central point influent the price and the reliability of entire system.

In multi droppings system all subsystem control are slave and can communicate thought serial line only when the muster ask (in our case the central point). All subsystems can read the serial line bat only the sub system called can answer.

The central point scans all sub systems and display centralized data. Every data request includes the address of slave. The slave "feel" own address and an answer at the muster request. Every sub system has own address.

In this case all systems must run properly otherwise the chain is broken and the communication is impossible. A single malfunction cut down all communications. To avoid general brake down is necessary to use serial lines multiplexer.

A large quantity of cable is necessary bat entire system became more reliable.

The connecting way using serial line multiplexer is shown bellow

The serial multiplexer is made using a lot of serial interface circuits like IC 232 connected in a single serial line.

All outputs drivers are connected in a single point through an open collector driver. We achieve in this way an "or" logic function between all outputs.

The central computer sends data to multiplexer and the multiplexer send data for every sub system. The serial lines are separated and prevent the general brake down.

Figure 3

Figure 4 show a serial line multiplexer.

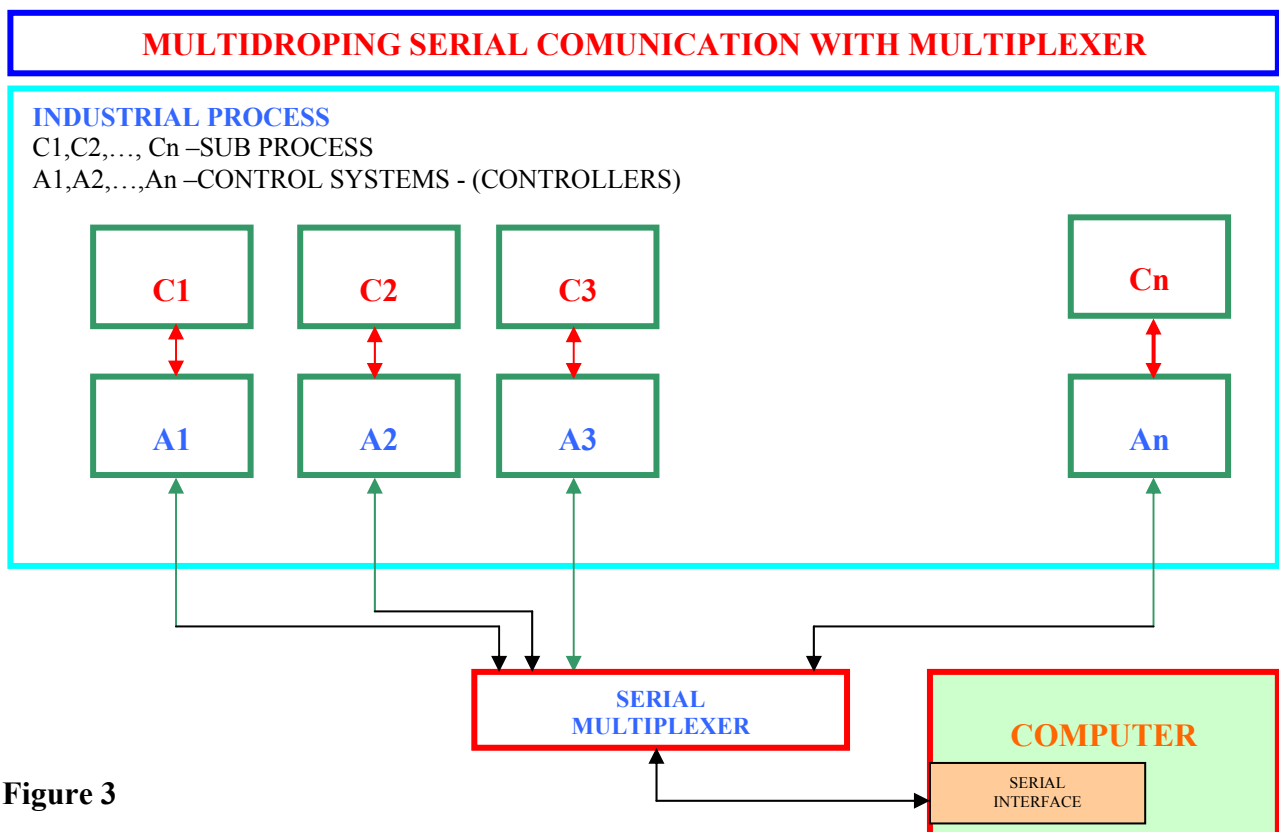**MULTIDROPING SERIAL MULTIPLEXER**

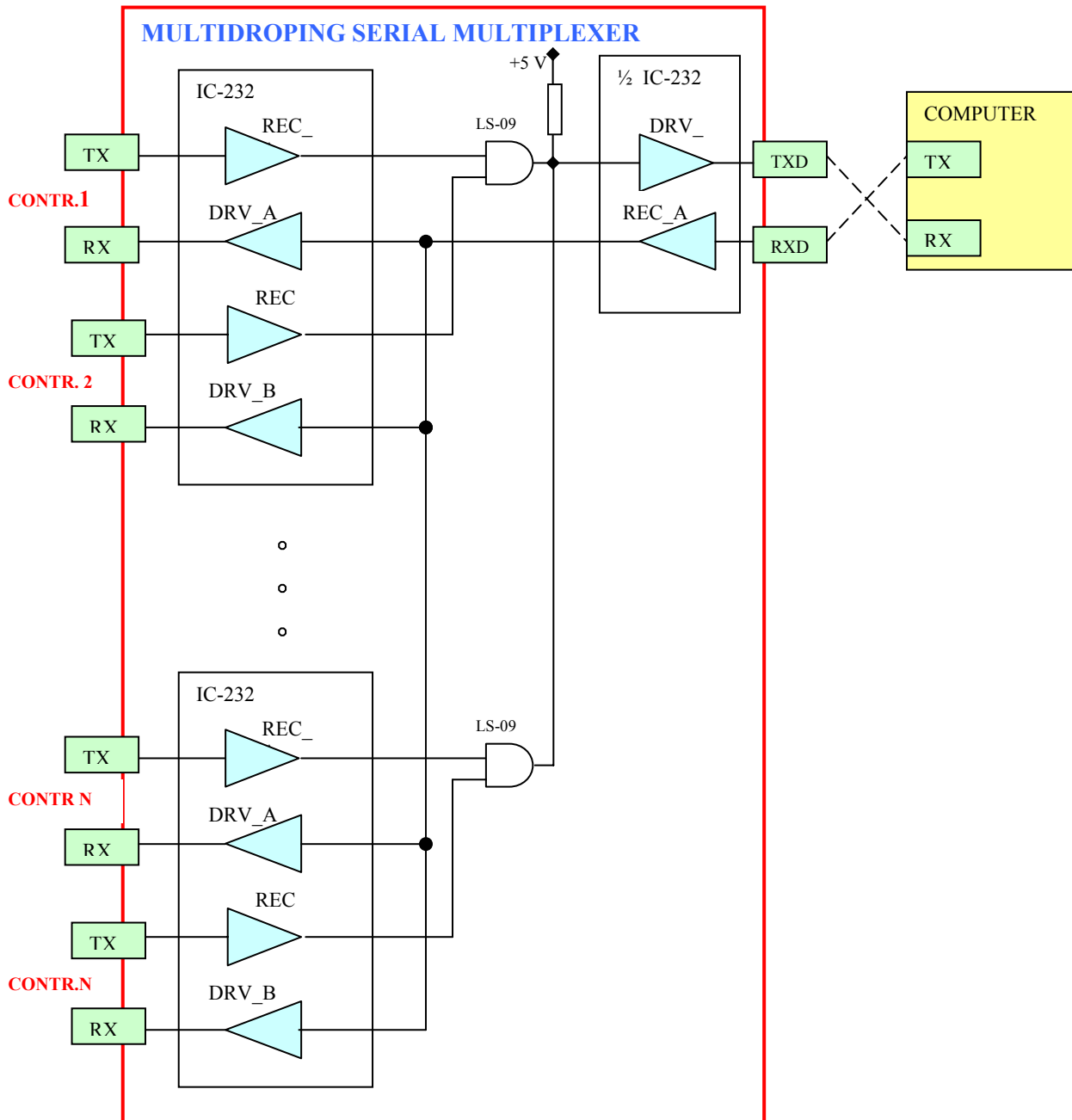**Figure 4**

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DISTRIBUTION TRANSPARENCY

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Abstract: *The SGBD system must appear like a centralized SGBD system. For that reason it must make a transparency series like: distribution transparency, transactions transparency, performances transparency. The transparency makes possible the existence of different SGBD into a single system.*

The key concern in the design of distributed systems is the preservation of transparency. In databases this means that each user can act as if the data was stored, managed and maintained on their own machine.

Transparency has a number of dimensions:

- location transparency;
- fragmentation transparency;
- replication transparency;
- failure transparency;
- concurrency transparency.

Distribution transparency allows the user to see the data base as a single logic entity. If a Distributed Database System has this transparency, then the user doesn't have to know that the data are fragmented (fragmentation transparency) or to know where these data are placed (location transparency).

To exemplify these concepts we assume that we have a relation:

Angajat (marcă, nume, prenume, adresă, nr_tel, filială, funcţie, sex, data_n, salariu, bi)

Vertically fragmented into:

S_1: $\Pi_{\text{marcă, funcție, sex, data\_n, salariu, bi}}$(Angajat) located on site 5

S_2: $\Pi_{\text{marcă, numee, prenume, adresă, nr\_tel, filiala}}$(Angajat) located on site 5

and S2 fragment horizontally fragmented according to the number of subsidiary:

S_{21}: $\sigma_{\text{filială= 'F3'}}$ (S2) located on site 3

S_{22}: $\sigma_{\text{filială= 'F5'}}$ (S2) located on site 5

S_{23}: $\sigma_{\text{filială= 'F7'}}$ (S2) located on site 7

Fragmentation is the highest level of transparency of the distribution that a Distributed Database System can secure. For finding the names of all directories, if fragmentation is transparent, we can write:

SELECT prenume, nume

FROM ANGAJAT

WHERE funcție= "Director"

In the case of location transparency, the users have to know the scheme of fragmentation, but not their location. An Oracle distributed database system has features that allow application developers and administrators to hide the physical location of database objects from applications and users. Location transparency exists when a user can universally refer to a database object such as a table, regardless of the node to which an application connects. Location transparency has several benefits, including:

- Access to remote data is simple, because database users do not need to know the physical location of database objects;

- Administrators can move database objects with no impact on end-users or existing database applications.

In this case the above interrogation becomes:

SELECT prenume, nume

FROM S_{21}

WHERE marcă IN (SELECT marcă FROM S1 WHERE funcție = 'Director')

UNION

SELECT prenume, nume

FROM S_{22}

WHERE marcă IN (SELECT marcă FROM S1 WHERE funcție = 'Director')

UNION

SELECT prenume, nume

FROM S$_{23}$

WHERE marcă IN (SELECT marcă FROM S1 WHERE funcţie = 'Director');

The main advantage of location transparency lies in the fact that the database can be reorganized physically, without having effects on application programs that they access.

There is the possibility that a certain fragment be replicated on several network computers. As a continuation of allocation transparency, Distributed Database System can secure replication transparency (reproduction transparency). It is possible for a system not to have location transparency, but reproduction transparency.

In the case of local transformations transparency, the user has to indicate both the names of the fragments and their location. In this case interrogation becomes:

SELECT prenume, nume

FROM S$_{21}$ AT SITE 3

WHERE marcă IN (SELECT marcă FROM S1 AT SITE 5 WHERE funcţie = 'Director')

UNION

SELECT prenume, nume

FROM S$_{22}$ AT SITE 5

WHERE marcă IN (SELECT marcă FROM S1 AT SITE 5 WHERE funcţie = 'Director')

UNION

SELECT prenume, nume

FROM S$_{23}$ AT SITE 7

WHERE marcă IN (SELECT marcă FROM S1 AT SITE 5 WHERE funcţie = 'Director')

In this case the key word AT SITE has been used to specify where a certain fragment is located. This interrogation is more complicated therefore it asks for more time than the previous two.

Apart from the distribution transparency there is the naming transparency. Distributed Database System has to guarantee that there won't be two sites to create two objects of database with the same name. One of the solutions, which could solve this problem, would be the creation of a central server name that has the role to guarantee the uniqueness of the names of the system. This solution could have negative effects such as:

- the loss of a part of local autonomy;
- problems of performance, if the central site would be overused;

- reduced availability – if the central site is not operational, the other sites cannot create new objects of database.

One solution for solving this problem can be attributing a prefix to each object that should contain the identifier of the site which created it, but this solution would lead to the loss of transparency at distribution. The solution for solving this problem would be the use o aliases for each object of database.

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FRAGMENTATION

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Abstract: *The fragmentation is an global collection logical operation for decompose in distributed data base into separate parts called fragments .The fragments are allocated to one or more sites.*

Fragmentation is the operation of logical decomposition of global collections, from a database which is distributed in disjunctive parts called fragments. SGBD makes the fragmentation by means of special operators applied to global collection. Fragmentation has to make the decomposition without loss of information and copies and to allow the reconstruction of the initial relation from fragments.

The essential argument in the favor of fragmentation lays in the possibility of performing operations in parallel, on different fragments. Therefore fragmentation increases competition. There are also arguments against fragmentation and they refer to the problems of semantic control and of data integrity.

For the database not to suffer semantic changes over the fragmentation process, the following rules are applied:

- fluffiness - if a relation R is decomposed in fragments R1, R2,…… Rn, then each data article from R can be found in one or more fragments. The application of this rule guarantees the fact that data are not lost during fragmentation;

- reconstruction- the possibility for recovering the relation R out of its fragments;

- disjunction- if an article of data d_1 appears in the fragment R_1, then it should not appear in any other fragment. Vertical fragmentation represents the exception from this rule and its basic key attributes should be repeated to allow reconstruction, thus the minimal redundancy of data is guaranteed.

Two fundamental strategies of fragmentation have been developed:

- Horizontal fragmentation- distributes a relation over its rows; so each fragment contains a subset from the relation's rows;

- Vertical fragmentation- produces from the relation R, the segments R1, R2,......Rn, each containing a subset of R's attributes, including primary key.

Horizontal fragmentation

Horizontal fragmentation of a relation is made through selections of global relation and is based on the use of predicates that impose a restriction on the relation's rows.

To present the horizontal fragmentation we refer to the following global relation:

ANGAJAT (marcă, nume, pnume, salar, impozit, funcție, nrdept)

Represented by rows as it follows:

Marcă	Nume	Pnume	Salar	Impozit	Funcție	Nrdept
m1	Pop	Ioan	300	15%	Inginer	1
m2	Stan	Mihai	250	10%	Economist	2
m3	Conț	Elena	400	20%	Analist	1
m4	Man	Anda	250	10%	Inginer	1
m5	Pascu	Andrei	300	15%	Operator	2

Table 1

The application within a department refers to its employees with a higher probability than to the employees of other departments. The horizontal fragmentation can be defined by the selection within a fragment of the rows referring to the employees that work in the same department. To give an example we can assume that there are only two departments and after the horizontal fragmentation we have the following fragments:

ANGAJAT1

Marcă	Nume	Pnume	Salar	Impozit	Funcție	Nrdept
m1	Pop	Ioan	300	15%	Inginer	1
m3	Conț	Elena	400	20%	Analist	1
m4	Man	Anda	250	10%	Inginer	1

Table 2

ANGAJAT2

Marcă	Nume	Pnume	Salar	Impozit	Funcție	Nrdept
m2	Stan	Mihai	250	10%	Economist	2
m5	Pascu	Andrei	300	15%	Operator	2

Table 3

The two resulting fragments can be marked symbolically as it follows:

ANGAJAT1 = $\sigma_{nrdept=1}$(ANGAJAT)

ANGAJAT2 = $\sigma_{nrdept=2}$(ANGAJAT)

Horizontal fragmentation can be:

- primary horizontal fragmentation that is made by the use of predicates defined on that relation;
- derived horizontal fragmentation that is made by the use of predicates defined on other relations.

For choosing the type of fragmentation it is necessary to analyze information related to data base and those related to the applications of data base. The analysis involves the examination of predicates or of the searching conditions used by transactions or queries within application.

Vertical fragmentation

The vertical fragmentation of a relation produces the fragments R1, R2,……Rn each of them containing a subgroup of attributes together with the primary key of R. Through vertical fragmentation, the attributes used by some applications are grouped, indicating the "affinity" of attributes. Affinity shows how tight the attributes are. One way to do this is to create a matrix that should show the number of accesses referring to each pair of attributes. One matrix is created for each transaction, as well as a general matrix that should represent the sum of all accesses for each pair of attributes. The pairs with higher affinity should be in the same vertical fragment and the pairs with less affinity can appear separately.

This action is called division. As a result of its use a set of fragments, that don't overlap, is obtained, thus generating the disjunctive character. This characteristic is applied only to the attributes that don't belong to the primary key.

To exemplify these concepts we assume that we have a relation:

Angajat (marcă, nume, pnume, adresă, nr_tel, filială, funcție, sex, data_n, salariu)

vertically fragmented into:

S_1: $\Pi_{marcă, funcție, sex, data\_n, salariu}$(Angajat)

S_2: $\Pi_{\text{marcă, nume, pnume, adresă, nr\_tel, filiala}}(\text{Angajat})$

Through this, two fragments are produced. Each contains the primary key mark, to allow the reconstruction of the initial relation

Fragment S1

Marcă	Funcție	Sex	Data_n	Salariu
m1	Inginer	M	10.02.56	300
m2	Economist	M	26.04.57	450
m3	Analist	F	12.11.59	500
m4	Inginer	F	08.06.86	350
m5	Operator	M	19.01.81	400

Table 4

Fragment S2

Marcă	Nume	Pnume	Adresă	Nr_tel	Filială
m1	Pop	Ioan	Xx	X	2
m2	Stan	Mihai	Zz	Y	1
m3	Conț	Elena	Yy	Z	1
m4	Man	Anda	Tt	T	2
m5	Pascu	Andrei	Nn	N	3

Table 5

Mixed fragmentation

Mixed fragmentation, also called hybrid involves the decomposing of a global collection into fragments through the successive application of horizontal and vertical methods. Generally, a vertical fragmentation can be followed by a horizontal one and vice versa, having as a result a structured partitioning in the shape of a tree.

Mixed fragmentation is defined with the help of the operations of selection and projection from relational algebra.

The number of the fragmentation levels can be big, yet it is a finite number because in the case of horizontal fragmentation it finishes when a fragment is made up of a single row, and in the case of vertical fragmentation it stops when a fragment is made up of an attribute. In practice, the level of fragmentation is not higher than tow because of the cost.

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